January 27, 2004

Introduction.

from "An open system Approach to Quantum Optics" H. Carmichael

Open systems. → invention of the laser

↳ dissipation.

Sources of light are open systems.

The quantum theory of the laser needed a way to treat dissipation in a quantum mechanical way.

The central ideas of a dissipative process up to then were in Fermi's golden rule. It gives us a picture of quantum dynamics expressed in terms of transitions between discrete states.

However, the fundamental equations of quantum mechanics are not expressed in these terms; they describe the continuous evolution of complex amplitudes, the coefficients in a superposition of states.

Since the laser is a coherent source, the laser dynamics involves complex amplitudes. We have to go beyond the calculation of rates for the incoherent emission and absorption of photons. The quantum theory of the laser has to reveal both the coherence and incoherence in open system dynamics.

Calculations in quantum optics for open systems follow two principal methods that are somehow a consistent language for the matter:

i) ~~Quantum~~ Langevin eq. based on the Heisenberg equations of motion.

ii) Master equation rooted in the Schrödinger or wavefunction picture.

We will focus more on the master equation but we will need to look at Heisenberg equations as they are the ingredient for multi-time averages or correlation functions, of the fields radiated.

The laser introduced a novelty in QED: the field inside a laser cavity involves a very large number of photons; traditionally QED dealt with perturbative calculations on a few photons at most.

Glauber developed the language to describe laser light in terms of highly excited states of a quantum field, based on the idea of the coherent state as the closest analogue of a classical field of fixed amplitude and phase

From the coherent state language of Glauber many phase-space methods have followed. They allow the transformation of a an operator master equation to be rewritten to look like a classical Fokker-Planck equation with a drift (drive) and diffusion (dissipation) part.

Most quantum states do not possess the simple form of coherent state representation that underlies the dynamical picture based on classical diffusion. The Glauber P ~~[illegible]~~ distribution represents mixtures of coherent states; appropriate for conventional lasers that do radiate mixtures of coherent states. However, non-linear interactions convert laser light into something other than a mixture of coherent states. In these situations the Fokker Planck equation is not useful.

These states, generated through nonlinear interactions, appear in the study of antibunching, sub-Poissonian statistics, and squeezing.

The experiments that have forced a new look at the open system methods come from cavity QED.

The whole objective in cavity QED is to achieve experimental conditions that make the ~~[illegible]~~ field of just one photon large, where "large" is measured with respect to the same intrinsic nonlinearities that turn coherent states into something else. So the approach following Glauber of "many photons" do not work.

Howard Carmichael realized this problem and developed a different approach starting in the mid eighties. The new approach employs "quantum trajectories". This "quantum trajectory" refers to the path of a stochastic wavefunction that describes the state of an optical source, conditioned at each instant on a history of classical stochastic signals realized at ideal or nonideal detectors monitoring the fields radiated by the source.

This approach is extremely useful to interact with the experiments. Most quantum optics experiments involve sources of light that are photoemissive.

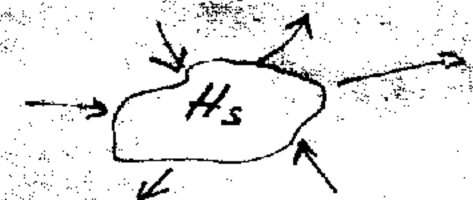
The sources emit photons irreversibly, to propagate away from the source until they are absorbed in the walls of the laboratory or are detected. Contrast this with the idealized picture of an electromagnetic field confined within a perfect cavity and measured inside the cavity by a detector introduced into that otherwise lossless environment. In the first scenario the act of detecting photons does not directly interfere with the source (more about this later in the course)

since the photons have already left the source, irreversibly, before they encounter the detector.
In the second, the detector is a major intrusion; the cavity field dynamics when the detector is present must be quite different from the description when it is not.

Here you can see why Cavity QED would be quite different → the "atom" is the detector inside the cavity! One photon is a major perturbation

In what follows we will look briefly at the approaches to dissipation in Quantum Optics.

We need to consider in very general terms the dynamics of a Harmonic Oscillator = system when we couple it to the external world. We need to do so in order to measure it or to drive it.



So far we have considered pure (closed) systems. now we will open them

From Statistical Mechanics fluctuation-dissipation theorem we expect changes in the behavior but we have to keep the Quantum Mechanics consistent.

Let us start with an ad-hoc approach.

in classical mechanics the essential features of dissipation: the decay of the oscillator amplitude can be built into the theory by the simple addition of a velocity dependent force.

Take the H.O.

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 q^2$$

the equations of motion

$$\frac{\partial H}{\partial p} = \dot{q} = \frac{p}{m} \qquad -\frac{\partial H}{\partial q} = \dot{p} = -m\omega^2 q$$

then it becomes a damped ~~harmonic~~ oscillator with the addition of a force $-\gamma p$

which modifies the eq. of motion

$$\dot{q} = \frac{p}{m}$$

$$\dot{p} = \underline{-\gamma p} - m\omega^2 q$$

And then one gets the familiar equation.

$$\ddot{q} + \gamma\dot{q} + \omega^2 q = 0$$

→ see next page (3a)

Can we simply transform this approach to the quantized H.O.

now $\hat{p}$ and $\hat{q}$ are operators. The above equations.

$$\dot{\hat{q}} = \frac{\hat{p}}{m}$$

$$\dot{\hat{p}} = -\gamma\hat{p} - m\omega^2\hat{q}$$

are the Heisenberg equations of motion

$$[\hat{q}, \hat{p}] = i\hbar$$

The equations remain linear.

The classical solutions still hold.

The expectation values of $\langle\hat{p}\rangle$ and $\langle\hat{q}\rangle$ will be damped in the same way as the classical variables, we seem to be in good shape.

However look at the evolution of the commutator $[\hat{q}, \hat{p}]$

$$\frac{d}{dt}[\hat{q}, \hat{p}] = \dot{\hat{q}}\hat{p} + \hat{q}\dot{\hat{p}} - \dot{\hat{p}}\hat{q} - \hat{p}\dot{\hat{q}}$$

although the above procedure is often adequate in C.M. even there it provides an incomplete description of dissipation. The eq. of motion for the H.O. before "dissipation" are time-reversal invariant, while

$$\dot p = -\gamma p - m\omega^2 q$$

the time-reversal has been broken!

If we want to understand the origin of this irreversibility we must begin with the recognition that the oscillator is damped through interactions with a large and complicated system: its environment. There is a fundamental relationship between fluctuation and dissipation. If the environment is some large system in thermal equilibrium, it will exert a fluctuating force $F(t)$ on an oscillator coupled to it in addition to soaking up the oscillator's energy.

Equation: $\ddot q + \gamma \dot q + \omega^2 q = 0$ for the familiar force has to have a drive term $\frac{F(t)}{m}$

and now one gets a stochastic differential equation.

In many situations the added noise source can not be overlooked - the most important technologically relevant examples are electrical circuits.

$$\frac{d}{dt}[\hat{q},\hat{p}] = \cancel{\frac{\hat{p}^2}{m}} - \gamma\hat{q}\hat{p} - \cancel{m\omega^2\hat{q}^2} + \gamma\hat{p}\hat{q} + \cancel{m\omega^2\hat{q}^2} - \cancel{\frac{\hat{p}^2}{m}}$$

$$= -\gamma[\hat{q},\hat{p}]$$

so that the solution:

$$[\hat{q}(t),\hat{p}(t)] = e^{-\gamma t}[\hat{q}(0),p(0)] = e^{-\gamma t}\, i\hbar$$

Then the Heisenberg uncertainty

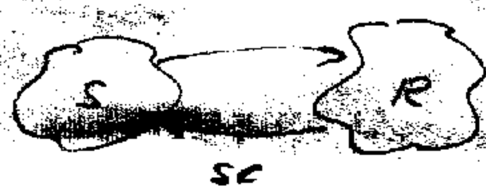
$$\Delta q\,\Delta p \geq \frac{1}{2}\hbar e^{-\gamma t}$$

evolves in time and decreases to zero.

There is a problem, we can not simply <u>add</u> a term, we have to add an interaction.

First look at the interaction of two H.O.

$$\hat{H} = \hat{H}_S + \hat{H}_R + \hat{H}_{SR}$$



$$\hat{H}_S = \hbar\omega_0\hat{a}^+\hat{a}$$
$$\hat{H}_R = \hbar\omega_0\hat{b}^+\hat{b}$$

with $\hat{a}^+, a$; $\hat{b}^+, b$ boson operators.

$$\hat{H}_{SR} = \hbar\underbrace{\bar{\kappa}}_{\text{coupling constant}}(\hat{a}^+\hat{b} + a\hat{b}^+)$$

[Rotating wave approximation]

to solve this problem let us introduce the normal-mode operators.

$$\hat{C} = \frac{\hat{a}+\hat{b}}{\sqrt{2}} \qquad \hat{D} = \frac{\hat{a}-\hat{b}}{\sqrt{2}}$$

$$\hat{H} = \hbar(\omega_0 + \bar{k})\hat{C}^+\hat{C} + \hbar(\omega_0 - \bar{k})\hat{D}^+\hat{D}$$

(because there is exchange of ~~energy the~~ excitation the energy levels get shifted up and down the rate at which energy goes from the system to the reservoir).

The time evolution for the ~~normal-mode~~ follow from the Heisenberg equation of motion.

$$\dot{\hat{C}}(t) = \frac{1}{i\hbar}[\hat{C}(t), \hat{H}]$$

$$\dot{\hat{C}}(t) = -i(\omega_0 + \bar{k})C(t)$$

$$\hat{C}(t) = C_0 e^{-i(\omega_0 + \bar{k})t}$$

and in a simmilar way

$$\hat{D}(t) = D_0 e^{-i(\omega_0 - \bar{k})t}$$

there is the familiar mode splitting of the eigenfunctions.

then back to the original modes:

$$\hat{a}(t) = [\hat{a}_0 \cos\bar{k}t - i\hat{b}_0 \sin\bar{k}t]e^{-i\omega t}$$

$$\hat{b}(t) = [\hat{b}_0 \cos\bar{k}t - i\hat{a}_0 \sin\bar{k}t]e^{-i\omega t}.$$

the initially independent operators $a(t=0) \equiv a_0$ $b(t=0) = b_0$ become mixed as a result of their interaction.

The system commutator

$$[\hat{a}(t), \hat{a}^+(t)] = [\hat{a}_0, \hat{a}_0^+] \cos^2 \kappa t + [b_0 \hat{b}^+] \sin^2 \kappa t$$

$$= 1$$

it is preserved only by mixing in properties of the reservoir b ~~mode~~.

We need the operator character of the reservoir.

Jan, 29 2004

to obtain ~~the~~ ~~...~~ [illegible] of the ... [illegible]