Solution to A

A. They are the same. In the basis $\{|\alpha>\}$ that diagonalizes V, $V|\alpha>= v_\alpha|\alpha>$ so $(H_0 + \lambda V)|\alpha>= (E^{(0)} + \lambda v_\alpha)|\alpha>$

Going the other way, if $H|\alpha>= E_\alpha|\alpha>$, then $\lambda V|\alpha>= (H - H_0)|\alpha>= (E_\alpha - E_D^{(0)})|\alpha>= \Delta_\alpha|\alpha>$, so $|\alpha>$ is also an eigenvector of V.