A. Are the states that diagonalize V in the degenerate subspace the same as the states that diagonalize $H_0 + \lambda V$?

Either prove that they are or give a counter-example.