Lecture $\boxed{2}$ (Sep. 2, Wed.)

$\boxed{\text{Outline}}$: use Stern-Gerlach (SG) experiment to motivate setting-up formalism / language for QM, such as how to describe state of a system and observables corresponding to that system ("kinematics"); then dynamics (Schroedinger equation)

Each Ag atom has 1 unpaired e^- : only spin

$\bar{\mu} = \bar{S}\left(\frac{e}{m_e c}\right)$ ... $F_z = \mu_z \partial B_z / \partial z$
$\propto S_z$

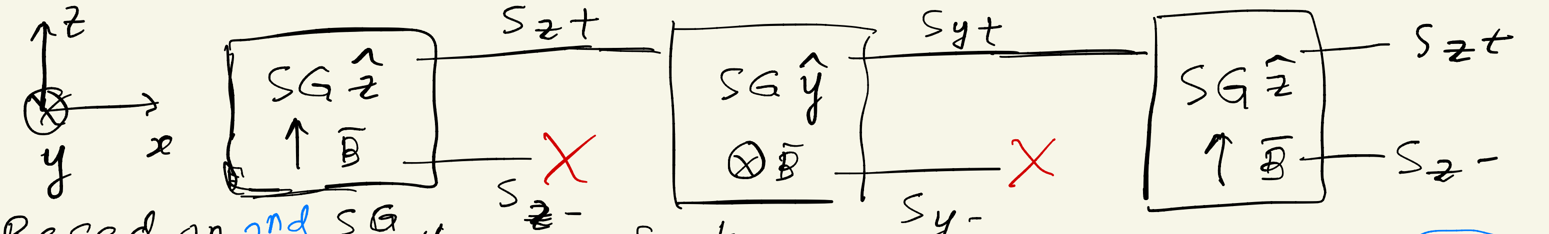
deflection measures S_z

classically : spins oriented randomly ... $\Rightarrow$ band

.... (1). S_z quantized $= \pm \hbar/2$ $(S_z \pm)$

with B_y : $S_y \pm$ (operators have discrete eigenvalues)

(2) Multiple SG (see HW 2.2)



Based on 2nd SG result

~~interpret (?)~~ : 50% of atoms out of SG $\hat{z}$ are S_z+ & S_y+;

other 50% S_z+ & S_y-

based on 3rd SG result

$\Rightarrow$ S_z & S_y can't be determined simultaneously → cf. classical e.g. L_z & L_x for spinning top

(operators don't commute)

— Spin state of Ag atom is $\boxed{\text{vector}}$ in abstract, 2D space

$|S_z;\pm\rangle$ base vectors

due to result of 3rd SG

$$|S_y;+\rangle \stackrel{?}{=} \frac{1}{\sqrt{2}}|S_z;+\rangle + \frac{1}{\sqrt{2}}|S_z;-\rangle$$

$$|S_y;-\rangle \stackrel{?}{=} \frac{1}{\sqrt{2}}|S_z;+\rangle - \frac{1}{\sqrt{2}}|S_z;-\rangle$$

orthogonality

— By symmetry, $|S_x;\pm\rangle$: "similar" to $|S_y;\pm\rangle$, but no "room"

... not quite : $|S_x;\pm\rangle \stackrel{?}{=} \frac{1}{\sqrt{2}}|S_z;+\rangle \pm \frac{i}{\sqrt{2}}|S_z;-\rangle$

$\Rightarrow$ QM-states represented by vectors in abstract, complex space

(I). Ket space : complex, abstract vector space (VS)

e.g. 2 D for spin of Ag atom (spin-½) ; ∞ dimensional (position / momentum)

$|\alpha\rangle$ (ket, vector in this space), e.g. $|S_z; \pm\rangle$

$|\alpha\rangle + |\beta\rangle = |\gamma\rangle$; $c|\alpha\rangle$ represent same state

c ↑ complex number

$$|S_y; +\rangle = \frac{1}{\sqrt{2}}|S_z; +\rangle + \frac{1}{\sqrt{2}}|S_z; -\rangle$$

(II) Bra (dual to ket) space : $\langle\alpha| \longleftrightarrow |\alpha\rangle$

$$|\alpha\rangle + |\beta\rangle \longleftrightarrow \langle\alpha| + \langle\beta|$$

$$c|\alpha\rangle \longleftrightarrow c^*\langle\alpha|$$

Inner product : $\langle\beta|\alpha\rangle = (\langle\beta|)\cdot(|\alpha\rangle)$

Properties

(a) $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^* \Rightarrow \langle\alpha|\alpha\rangle$ real

(b) $\langle\alpha|\alpha\rangle > 0$ unless $|\alpha\rangle$ is null

probabilistic interpretation in QM

} postulates

(c) $\langle\alpha|\beta\rangle = 0$ orthogonal

(d) Normalized ket : $|\tilde{\alpha}\rangle = \dfrac{|\alpha\rangle}{\sqrt{\langle\alpha|\alpha\rangle}}$