Phys 601 Dr. Agashe

(very) Last step in proof of parallel axes theorem

(see top of page 8 of dynamics note)

— We would like to show that

$$\sum_{i} m_i \, (r_i)_a = 0 \quad \ldots (1)$$

points/or particles in body → i

a ↳ given, i.e., $a = 1$ or 2 or 3 (x or y or z)

r_i: position vector of i^{th} point relative to COM (denoted by P in this proof)

— Begin with origin being at an arbitrary point instead. Let $\bar{r}_{i,\text{"old"}}$ denote position vector of i^{th} point and $\bar{c}$ that of COM with respect to this origin.

— Clearly, definition of COM gives

$$c_a = \left[\sum_i m_i \, (r_{i,\text{old}})_a \right] \Big/ \left(\sum_i m_i \right) \quad \cdots (2)$$

and new and old position vectors are related by:

$$\bar{r}_i = \left(\bar{r}_{i,\text{old}} - \bar{c} \right) \quad \cdots (3)$$

$\bar{r}_i$: COM as origin; $\bar{r}_{i,\text{old}}$: ↳ arbitrary origin; $\bar{c}$: ← independent of $\boxed{i}$

— Plugging Eqs. (2) & (3) on LHS of Eq. (1) gives

$$\sum m_i \, (r_{i,\text{old}})_a - \left(\cancel{\sum m_i} \right) \sum_i m_i (r_{i,\text{old}})_a \Big/ \left(\cancel{\sum m_i} \right)$$

$$= 0 \text{ as desired}$$