Here is the derivation of Lorentz force using the Hamiltonian formalism (following DT's example 2 in section 4.1.3). Start with the Lagrangian that we used before, i.e., in terms of scalar (ϕ) and vector ($\mathbf{A}$) potentials

$$L = \frac{1}{2}m|\dot{\mathbf{r}}|^2 - e\left(\phi - \dot{\mathbf{r}}.\mathbf{A}\right) \tag{1}$$

so that momentum conjugate to the position is given by

$$\mathbf{p} = \frac{\partial L}{\partial \dot{\mathbf{r}}} = m\dot{\mathbf{r}} + e\mathbf{A} \tag{2}$$

inverting which gives

$$\dot{\mathbf{r}} = \frac{1}{m}\left(\mathbf{p} - e\mathbf{A}\right) \tag{3}$$

Note that conjugate momentum is *not* entirely the mechanical momentum, which would be just $m\dot{\mathbf{r}}$. So, Hamiltonian is obtained as

$$\begin{aligned} H() &= \mathbf{p}.\dot{\mathbf{r}} - L \\ &= \frac{1}{m}\mathbf{p}.\left(\mathbf{p} - e\mathbf{A}\right) - \left[\frac{1}{2m}\left(\mathbf{p} - e\mathbf{A}\right)^2 - e\phi + \frac{e}{m}\left(\mathbf{p} - e\mathbf{A}\right).\mathbf{A}\right] \\ &= \frac{1}{2m}\left(\mathbf{p} - e\mathbf{A}\right)^2 + e\phi \end{aligned} \tag{4}$$

(Note that both the potentials could have explicit time-dependence; in addition, they depend on position of the charged particle, which itself is changing with time.) This gives Hamilton's equations:

$$\dot{\mathbf{r}} = \frac{\partial H}{\partial \mathbf{p}} = \frac{1}{m}\left(\mathbf{p} - e\mathbf{A}\right) \tag{5}$$

and (in component form)

$$\dot{p_x} = -\frac{\partial H}{\partial x} = -e\frac{\partial \phi}{\partial x} + \frac{e}{m}\left(p_i - eA_i\right)\frac{\partial A_i}{\partial x} \tag{6}$$

where in the last term i is summed over x, y and z. Using Eq. (5) in last term of Eq. (6), we get

$$\dot{p_x} = -e\frac{\partial \phi}{\partial x} + ev_i\frac{\partial A_i}{\partial x} \tag{7}$$

where v_i's are components of the *velocity* of the particle, i.e., $\dot{\mathbf{r}}$. We can take another time derivative of LHS of Eq. (5) to give the force:

$$\mathbf{F} = m\ddot{\mathbf{r}} = \dot{\mathbf{p}} - e\frac{d\mathbf{A}}{dt} \tag{8}$$

Now, in component form

$$\begin{aligned}\frac{dA_x(\mathbf{r},t)}{dt} &= \frac{\partial A_x}{\partial t} + \frac{\partial A_x}{\partial x}\frac{dx}{dt} + \frac{\partial A_x}{\partial y}\frac{dy}{dt} + \frac{\partial A_x}{\partial z}\frac{dz}{dt} \\ &= \frac{\partial A_x}{\partial t} + \frac{\partial A_x}{\partial x}v_x + \frac{\partial A_x}{\partial y}v_y + \frac{\partial A_x}{\partial z}v_z \qquad (9)\end{aligned}$$

Plugging Eqs. (7) and (9) into x-component of RHS of Eq. (8), and collecting/cancelling terms, gives

$$\begin{aligned}F_x &= -e\left(\frac{\partial \phi}{\partial x} + \frac{\partial A_x}{\partial t}\right) + +e\Big[v_y\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + v_z\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\Big] \\ &= eE_x + e\Big[v_y\left(\nabla\times\mathbf{A}\right)_z - v_z\left(\nabla\times\mathbf{A}\right)_y\Big] \qquad (10) \\ &= eE_x + e\left(v_yB_z - v_zB_y\right) \qquad (11) \\ &= eE_x + e\left(\mathbf{v}\times\mathbf{B}\right)_x \qquad (12)\end{aligned}$$

where we used $\mathbf{E} = -\nabla\phi$ in getting to the 1st term in Eq. (10) and $\mathbf{B} = \nabla\times\mathbf{A}$ in last 2 terms in Eq. (11), ending up with the usual formula for the (x-component of the) Lorentz force acting on a charged particle moving in electric and magnetic fields.