PHYS420 (Spring 2002)
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Homework # 9
(due Monday, April 22nd , 2002)

Problems

1. **(10 points) Wavelengths**. What is the approximate wavelength of
   (a) a car moving at 60 miles per hour?
   (b) a cell moving at 1mm per hour?
   (c) an electron with an energy of 10 eV?
   (d) a photon with an energy of 10 eV?
   (e) a neutron with an energy of 0.1 eV?

Answer: (a) Assuming a car mass somewhere between 1500 – 3000 lbs. In MKS units this would correspond to a typical mass of about 1000 kilograms (about 1 metric t on). If the car is moving at 60 miles per hour, in MKS units, this is

$$v = 60\frac{miles}{hour}(\frac{5280\,ft}{mile})(\frac{12\,in}{ft})(\frac{0.0254m}{in})(\frac{1\,hour}{3600\,\sec}) = 26.8\,m/s$$

The momentum (non-relativistic) is thus:

$$p = mv = (1000\,kg)(26.8\,m/s) = 2.68x10^{4}\ kg\ m/s.$$

According to de Broglie, the wavelength is thus:

$$\lambda = \frac{h}{p} = \frac{6.626x10^{-34}\,J\cdot s}{2.68x10^{4}\,kg\cdot m/s} = \mathbf{2.47x10^{-38}\ m}$$

Of course, your numbers may vary somewhat from this if you choose a different mass for the car.

(b) Cells vary greatly in size and shape, but to a physicist a typical cell could be well represented by a sphere of water with a radius of a few µm's (let's say 2.5 µm). We can find the mass of the cell by multiplying the volume of the cell time its density ($\rho_{water} = 1\,gm/cm^3$). Putting all the numbers together in consistent units:

$$m = \rho_{water}Vol = \rho_{water}(\frac{4}{3}\pi r^3) = (10^3\,\frac{kg}{m^3})\frac{4}{3}\pi(2.5x10^{-6}\,m)^3 = 6.54x10^{-14}\ kg$$

$$v = 1\frac{mm}{hour} = 2.77x10^{-7}\ m/s$$

The de Broglie wavelength is thus:

$$\lambda = \frac{h}{mv} = \frac{6.626x10^{-34}\,J\cdot s}{(6.54x10^{-14}\,kg)(2.77x10^{-7}\,m/s)} = \mathbf{3.66x10^{-14} m.}$$

(c) Since the kinetic energy of the electron (10 eV) is much less than its rest energy (0.511 MeV), we can use non-relativistic momentum ($p = \sqrt{2mE}$).

$$[i.e.\ E = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2E}{m}}\ ;\ \ p = mv = m\sqrt{\frac{2E}{m}} = \sqrt{2mE}\ ]$$

For this case the de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2m_e E}} = \frac{6.626x10^{-34}\,J\cdot s}{\sqrt{2(9.11x10^{-31}\,kg)(10eV\cdot 1.602x10^{-19}\,J/eV)}} = 3.88x10^{-10}\ m$$

$$= \mathbf{3.88\ Å}.$$

(d) For a photon we can just use our energy frequency relation,

$$\lambda = \frac{hc}{E} = \frac{1239.7\,eV\cdot nm}{10\,eV} = \mathbf{124\ nm}.$$

(e) Using similar arguments as in (c),

$$\lambda = \frac{h}{\sqrt{2m_n E}} = \mathbf{0.91\ Å}.$$

2. Show that the de Broglie wavelength of an electron accelerated from rest through a small potential difference V is given by $\lambda = \frac{1.226}{\sqrt{V}}$, where λ is in nanometers and V is in volts.

Answer: The kinetic energy that an electron, starting from rest, receives from a potential difference of V, is simply $KE = eV$. *Therefore the received momentum is just* $p = \sqrt{2mKE} = \sqrt{2meV}$. *The de Broglie wavelength is given by*

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e e}}\frac{1}{\sqrt{V}}$$

$$\lambda = \frac{6.626x10^{-34}\,J\cdot s}{\sqrt{2(9.11x10^{-31}\,kg)(1.602x10^{-19}\,C)}}\frac{1}{\sqrt{V}}$$

$$\lambda = \frac{1.226x10^{-9}\,m\cdot volts}{\sqrt{V\,[volts]}} = \frac{1.226[nm]}{\sqrt{V\,[volts]}}$$

3. **SMM, Chapter 4, problem 22.**

Answer: According to Heisenberg's Uncertainty Principle: $\Delta x\,\Delta p \ge \hbar/2$. *For this case,* $\Delta p = m\Delta v = (0.05kg)(0.001*30\,m/s) = 1.5x10^{-3}\,kg\cdot m/s$

Therefore, the minimum uncertainty in the position is

$$\Delta x = \frac{\hbar}{2\Delta p} = \frac{\left(6.626x10^{-34}\,J\cdot s/2\pi\right)}{\left(2*1.5x10^{-3}\,kg\cdot m/s\right)} = \mathbf{3.51x10^{-32}\ meters}.$$

4. **SMM, Chapter 4, problem 27.**

Answer: This is an interesting example of what life would be like if quantum mechanical effects were large enough to be seen in macroscopic systems.
(a) The minimum uncertainty in Fuzzy's momentum is

$$\Delta p = \frac{\hbar}{2\Delta x} = \frac{(2\pi\, J \cdot s / 2\pi)}{(2 * 1m)} = \frac{1}{2} kg \cdot m/s$$

which gives a minimum speed of $\Delta v = \frac{\Delta p}{m} = \frac{1/2\ kg \cdot m/s}{2.0\,kg} =$ ***0.25 m/s****.*

*(b) Fuzzy might move by (0.25 m/s) * (5 s) = 1.25 m. With the original position uncertainty of 1 m, we can think of* Δx *growing to 1 m + 1.25 m =* ***2.25 meters****.*

5. **SMM, Chapter 4, problem 28.** *To keep this calculation as a general estimate, assume* $\Delta x \Delta p \approx \hbar$ *and that the momentum is roughly of the same order of magnitude as the uncertainty in the momentum (i.e.* $p \approx \Delta p$*).*

Answer: If you recall, this problem was fully worked out during lecture.

(a) Letting $\Delta x \approx r$*, we find that* $\Delta p \approx \frac{\hbar}{r}$*.*

(b) Estimating the kinetic energy: $KE = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \frac{\hbar^2}{2m_e r^2}$.

Estimating the potential energy: $PE \approx -\frac{ke^2}{r}$.

The total energy is just: $E_{total} \approx \frac{\hbar^2}{2m_e r^2} - \frac{ke^2}{r}$.

(c) To minimize E, we solve $\left.\frac{dE_{total}}{dr}\right|_{r=r_{\min}} = 0$ *for* r_{min}*:*

$$\left.\frac{dE}{dr}\right|_{r_{\min}} = \frac{ke^2}{r_{\min}^{\ 2}} - \frac{\hbar^2}{2m_e r_{\min}^{\ 3}} = 0$$

Solving for r_{min}*,*

$$r_{\min} = \frac{\hbar^2}{ke^2 m_e}$$

Which is exactly the Bohr radius! Therefore, it is not a surprise that when we plug this minimum radius into the total energy equation we get $E_{total} = -13.6$ *eV.*

6. **The width of spectral lines**. Although an excited atom can radiate at any time, the average time after excitation at which a group of atoms radiates is called the **lifetime**, τ. (a) If $\tau = 10$ nsec, use the uncertainty principle to compute the line width produced by this finite lifetime. (b) If the wavelength of the spectral line involved in this process is 500 nm, find the fractional broadening $\Delta f / f$.

Answer: This problem is an example from the book (Example 4.12, p. 170).
(a) We use $\Delta E\, \Delta t \approx \frac{\hbar}{2}$, *where* $\Delta E = h\Delta f$, *and where* $\Delta t = 1.0x10^{-8}\, s$ *is the average time available to measure the excited state. Thus,*

$$\Delta f = \frac{1}{4\pi x 10^{-8}\, s} = \mathbf{8.0x10^{6}\ Hz}$$

Note that ΔE *is the uncertainty in energy of the excited state. It is also the uncertainty in the energy of the photon emitted by an atom in this state.*
(b) First, we find the center frequency of this line as follows:

$$f_0 = \frac{c}{\lambda} = \frac{3.0x10^{8}\, m/s}{500x10^{-9}\, m} = 6.0x10^{14}\, Hz$$

Hence,

$$\frac{\Delta f}{f_0} = \frac{8.0x10^{6}\, Hz}{6.0x10^{14}\, Hz} = \mathbf{1.3x10^{-8}}$$

This narrow natural line width can be seen with a sensitive interferometer. Usually, however, temperature and pressure effects overshadow the natural line width and broaden the line through mechanisms associated with the Doppler effect and collisions.