PHYS420 (Spring 2002)
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Homework # 8 Solutions

Problems

1. The name *nuclide* is a general term for any isotope for any element. Using a table of nuclides, find all the possible *naturally occurring* isotopes of xenon (**Xe**) and cesium (**Cs**) along with their relative abundances. *An online table of nuclides can be found in the class website under 'Resources'.*

For xenon we have nine different naturally occurring isotopes.

Xe-124	*0.10 %*
Xe-126	*0.09 %*
Xe-128	*1.91 %*
Xe-129	*26.4 %*
Xe-130	*4.1 %*
Xe-131	*21.2 %*
Xe-132	*26.9 %*
Xe-134	*10.4 %*
Xe-136	*8.9 %*

For cesium we only have one.

Cs-133	*100 %*

2. Starting with the exponential law of radioactive decay, $N(t) = N_0 e^{-\lambda t}$ where λ is the decay constant, and using the definition of half-life ($T_{1/2}$), show that:

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

The half-life is defined as the time it takes half of a given number of radioactive nuclei to decay. Mathematically, this condition is defined as $N(T_{1/2}) = \frac{N_0}{2}$. *Plugging this is for N(t) and solving for $T_{1/2}$ yields the desired expression.*

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\ln(\frac{1}{2}) = -\lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

3. *The **decay rate** R, defined as the number of decays per unit time in a given radioactive sample, is given by* $R = \left|\frac{dN}{dt}\right|$. *The decay rate of a sample is often*

*referred to as its **activity** and frequently uses units of **curies** (Ci), defined as* $1Ci \equiv 3.7x10^{10}\, decays/s$. A sample of the isotope I^{131}, which has a half-life of 8.04 days, has an activity of 5 mCi at the time of shipment. Upon receipt of the I^{131} in a medical laboratory, its activity is 4.2 mCi. How much time has elapsed between the two measurements?

We calculate the decay rate using the definition,

$$R(t) = \left|\frac{dN(t)}{dt}\right| = N_0 \lambda e^{-\lambda t} = R_0 e^{-\lambda t}$$

Solving for t,

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R(t)}\right) = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{R_0}{R(t)}\right)$$

Plugging in the numbers,

$$t = \frac{8.04\, days}{\ln 2}\ln\left(\frac{5\, mCi}{4.2\, mCi}\right) = \textbf{2.02 days} \text{ have elapsed.}$$

4. The Actinium Series begins with the isotope $_{92}U^{235}$; each atom sends out in succession the following particles: α, β, α, β, α, α, α, α, β, β, α. From this information and by consulting the periodic table, write out an account of the Actinium Series (i.e. $_{92}\mathbf{U}^{235} \xrightarrow{\alpha} ? \xrightarrow{\beta} ? \ldots$) complete with the necessary symbols and the figures for atomic mass and atomic number. *Use the notation* $_ZX^A$, *where Z is the atomic number, X is the element name and A is the atomic weight to the nearest integer.*

According to the displacement law of radioactivity, when an atom undergoes alpha decay (i.e. helium nuclei), the atomic weight decreases by 4 and the atomic number decreases by 2. When the atom undergoes beta decay (i.e. electrons), the atomic weight doesn't change and the atomic number increases by 1. By applying these rules and using a periodic table we get the following series.

$$_{92}U^{235} \xrightarrow{\alpha} {}_{90}Th^{231} \xrightarrow{\beta} {}_{91}Pa^{231} \xrightarrow{\alpha} {}_{89}Ac^{227} \xrightarrow{\beta}$$
$$_{90}Th^{227} \xrightarrow{\alpha} {}_{88}Ra^{223} \xrightarrow{\alpha} {}_{86}Rn^{219} \xrightarrow{\alpha} {}_{84}Po^{215} \xrightarrow{\alpha}$$
$$_{82}Pb^{211} \xrightarrow{\beta} {}_{83}Bi^{211} \xrightarrow{\beta} {}_{84}Po^{211} \xrightarrow{\alpha} {}_{82}Pb^{207} \text{ (stable isotope)}$$

5. **Carbon-14 dating**. *Carbon-14 is a radioactive isotope (half life of 5730 years), created in the atmosphere by cosmic rays, that combines with oxygen to create carbon dioxide that is then absorbed by plants (and eventually makes it into plant eating organisms). Even though* C^{14} *is always decaying, it is continually being replenished in living things so that the ratio of* C^{14} *to normal carbon (*C^{12}*) is nearly constant. Once the organism dies, the* C^{14} *is no longer replenished and simply decays away. This decay can be used to determine how long ago the organism died.* The dirt floor of the Shanidar Cave in the northern part of Iraq has been examined. Below the layer of soil that contained arrowheads and bone awls

was a layer of soil that yielded flint tools and pieces of charcoal. When the charcoal was examined it was discovered that in 1 kg of carbon, approximately $9.4x10^2$ carbon-14 nuclei decayed each second. It is known that in 1 kg of carbon from living material, $1.5x10^4$ disintegrations of carbon-14 occur each second. Use this data to calculate when people of the Stone Age culture occupied the cave.

Since we are dealing with decays/sec, we can use the exponential formula for the decay rate (activity). Solving for t, as in problem 3, and plugging in the numbers, we get…

$$t = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{R_0}{R(t)}\right) = \frac{5730\,years}{\ln 2}\ln\left(\frac{1.5x10^4\,decays/s}{9.4x10^2\,decays/s}\right) = \boldsymbol{22898\ years}$$

6. **SMM, Chapter 3, problem 14.**

(a) The radii of the various Bohr orbits are given by $r_n = \frac{n^2 a_0}{Z}$ *where* $a_0 = 0.0529\,nm$. *Plugging Z = 1 (hydrogen) and n = 1, 2 & 3 we get…*

$r_1 = a_0 =$ ***0.0529 nm***
$r_2 = 4a_0 =$ ***0.2116 nm***
$r_3 = 9a_0 =$ ***0.4761 nm***

(b) The electron's speed is obtained by setting the Coulomb force equal to the centripetal force (SMM eq. 3.26): $v_n = \sqrt{\frac{ke^2}{m_e r_n}}$.

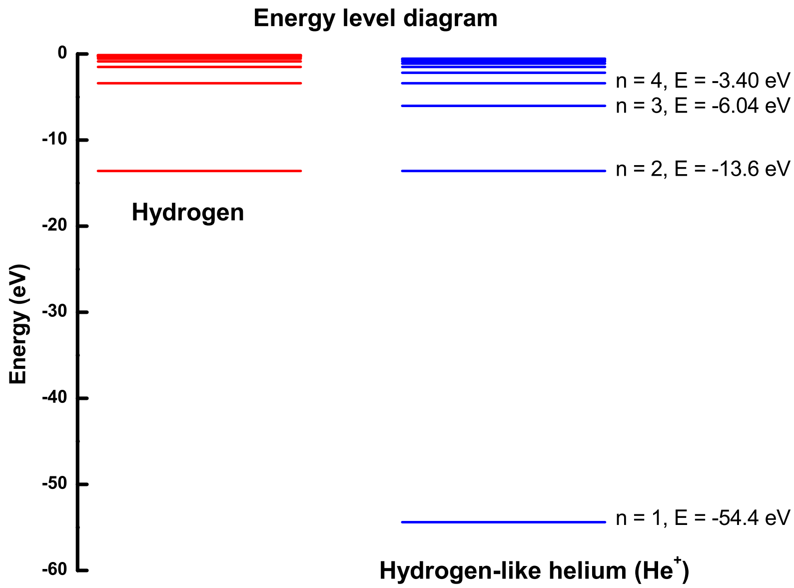
$$v_1 = \sqrt{\frac{(8.988x10^9\,Nm^2/C^2)(1.602x10^{-19}\,C)^2}{(9.109x10^{-31}\,kg)(0.0529x10^{-9}\,m)}} = \boldsymbol{2.19x10^6\ m/s} = 0.007c$$

$v_2 =$ $\boldsymbol{1.09x10^6\ m/s}$ $= 0.004c$
$v_3 =$ $\boldsymbol{7.28x10^5\ m/s}$ $= 0.002c$

(c) The velocities are much smaller than the speed of light. No relativistic correction is necessary.

7. **SMM, Chapter 3, problem 15.**

(a) The energy levels for a hydrogen-like ion whose charge number is Z is given by $E_n = -13.6\,eV\frac{Z^2}{n^2}$. *For helium (Z = 2), we plot an energy level diagram similarly to SMM Figure 3.23, where I've included the energy level diagram of hydrogen for comparison.*

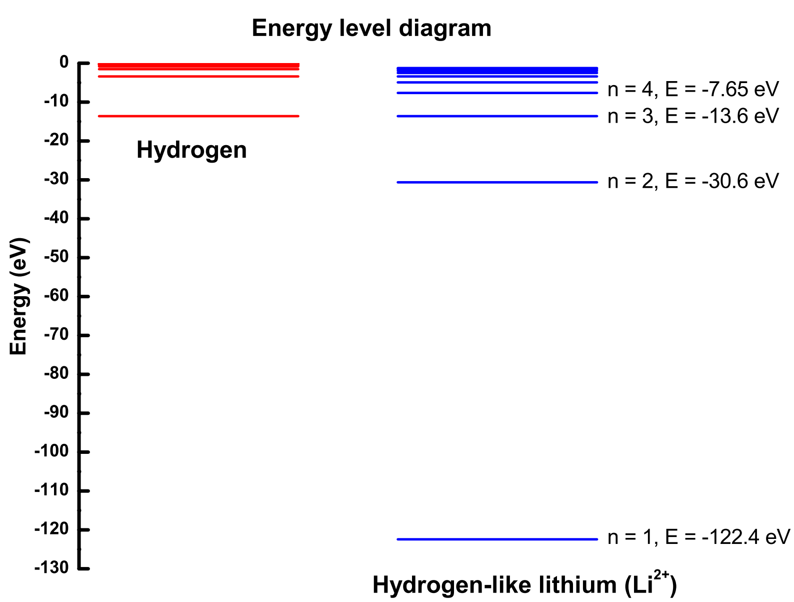


(b) The ionization energy is simply the energy required to take the electron from the state n = 1 to the state n = ∞. For hydrogen-like helium (He^{+}) this is just

$E_1 = -13.6\,eV\,\frac{2^2}{1^2} = \boldsymbol{-\,54.4\ eV}$.

8. **SMM, Chapter 3, problem 16.**

We follow the same procedure as in problem 7 but for hydrogen-like Lithium (Li^{2+}) which has Z = 3.



9. **SMM, Chapter 3, problem 17.**

Using $r_n = \dfrac{n^2 a_0}{Z}$ ...

(a) Z = 2 (Helium, He^{+}*),* $r_1 = \dfrac{(1)^2 a_0}{2} =$ ***0.0265 nm***

(b) Z = 3 (Lithium, Li^{2+}*),* $r_1 = \dfrac{(1)^2 a_0}{3} =$ ***0.0176 nm***

(c) Z = 4 (Beryllium, Be^{3+}*),* $r_1 = \dfrac{(1)^2 a_0}{4} =$ ***0.0132 nm***

Due to the increased positive charge in the nucleus, the first Bohr orbit gets smaller and smaller.