**PHYS420 (Spring 2002)**
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**Homework # 6 Solutions**

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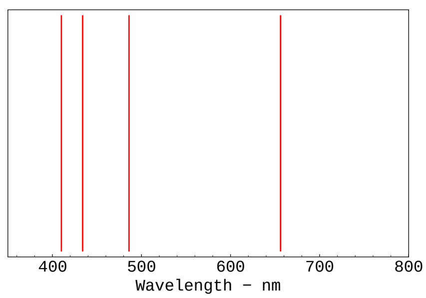
## Problems

1. The following is a plot showing the location of the first four lines of the Balmer series (i.e. 656, 486, 434 and 410 nm corresponding to $n_f = 2$ and $n_i = 3, 4, 5$ and 6). Using the Balmer – Rydberg formula, compute the location of the first four lines of the Lyman and Paschen series as well as their convergence limit. Make a similar graph showing all three series. Offset them vertically for clarity.




*A straightforward application of the Balmer – Rydberg formula,*

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \text{ where } R = 1.0973732 \times 10^7\ m^{-1},$$

*yields the following wavelengths for the Lyman and Paschen series. (To compute the convergence limit, set $n_i = \infty$).*

*For Lyman, $n_f = 1$ and $n_i = 2, 3, 4$ and $5$:*

$\lambda_{1\text{-}2}$ = ***121.5 nm***
$\lambda_{1\text{-}3}$ = ***102.5 nm***
$\lambda_{1\text{-}4}$ = ***97.2 nm***
$\lambda_{1\text{-}5}$ = ***94.9 nm***

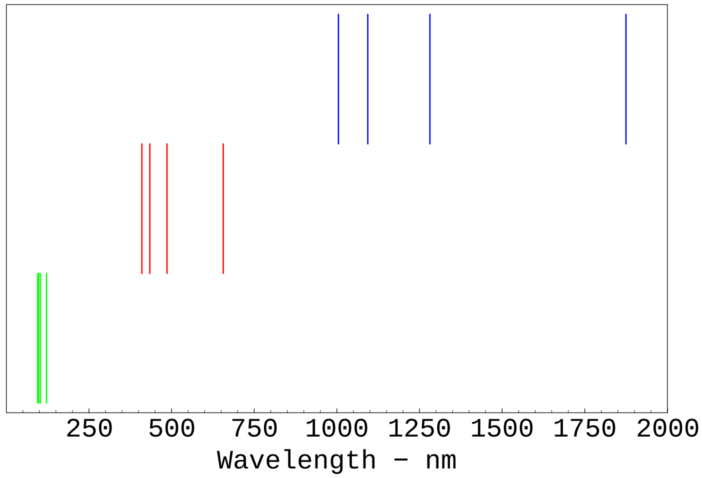
*with a convergence limit of $\lambda_{1\text{-}\infty}$ =* ***91.1 nm***

*For Paschen, $n_f = 3$ and $n_i = 4, 5, 6$ and $7$:*

$\lambda_{3\text{-}4}$ = ***1874.6 nm***
$\lambda_{3\text{-}5}$ = ***1281.5 nm***
$\lambda_{3\text{-}6}$ = ***1093.5 nm***
$\lambda_{3\text{-}7}$ = ***1004.7 nm***

*with a convergence limit of $\lambda_{3\text{-}\infty}$ =* ***820.1 nm***

*Plotting all three series together we get,*




2. Another interesting property of the hydrogen wavelengths is known as the *Ritz combination principle*. If we convert the hydrogen emission wavelengths to frequencies, we find the curious property that certain pairs of frequencies added together give other frequencies that appear in the spectrum! Show that the longest wavelength of the Balmer series and the longest two wavelengths of the Lyman series satisfy the Ritz combination principle.

*The longest wavelength of the Balmer series is 656.1 nm ($n_f = 2$, $n_i = 3$). Converting this into a frequency, we find* $f_{2-3} = c/\lambda_{2-3} = 4.57x10^{14}\,Hz$ *. The two longest wavelengths of the Lyman series are 121.5 nm ($n_f = 1$, $n_i = 2$) and 102.5 nm ($n_f = 1$, $n_i = 3$). Converting them into frequencies, we find* $f_{1-2} = 24.67x10^{14}\,Hz$ *and* $f_{1-3} = 29.24x10^{14}\,Hz$ *. Adding the smallest frequency of the Lyman series to the smallest frequency of the Balmer series gives the next smallest Lyman frequency:*

$$f_{1-2} + f_{2-3} = f_{1-3}$$
$$24.67x10^{14}\,Hz + 4.57x10^{14}\,Hz = 29.24x10^{14}\,Hz$$

3. **SMM, Chapter 3, Problem 3.**

*Since the initial deflection is downwards, we know that we are looking at a positive particle. Remember that in the configuration shown in the book, electrons were deflected upwards. J.J. Thomson's device works for positive and negative particles so we may apply,* $\frac{q}{m} = \frac{V\theta}{B^2 ld}$.

*(a)* $\frac{q}{m} = \frac{V\theta}{B^2 ld} = \frac{(2000 volts)(0.20 radians)}{(4.57x10^{-2}\,T)^2(0.10m)(0.02m)} =$ ***9.58 x 10^7 C/kg***

*(b) Since the particle is attracted by the negative plate, it carries a positive charge. We can guess that it's a* ***proton*** *(we don't really know of any other*

*particles right now). Checking...* $\frac{q_p}{m_p} = \frac{(1.602x10^{-19}C)}{(1.67x10^{-27}kg)}$ *= 9.59 x 10*$^7$ *C/kg. Close enough!*

*(c) The horizontal speed is given by the ratio of the Electric and Magnetic forces,*

$$v_x = \frac{E}{B} = \frac{V}{dB} = \frac{(2000volts)}{(0.02m)(4.57x10^{-2}T)} = \mathbf{2.19\ x\ 10^6\ m/s}$$

*(d) This is just* ***0.007c****. There is no need for relativistic mechanics.*

4. **SMM, Chapter 3, Problem 4 (10 points).**

*In this problem, we no longer have a magnetic field compensating for the deflection. The only force that our electrons feel is an electric force during the time spent between the parallel plates. During this time, which we'll call* $t_1$ *(equal to* $t_1 = \frac{l}{v_x}$*), the electron was subject to a force F = eE (providing an acceleration,* $a_1 = \frac{eE}{m}$*) and was deflected by a vertical amount* $y_1$*. Remember that the electric field for two parallel plates is* $E = \frac{V}{d}$*. This vertical deflection is just*

$$y_1 = \frac{1}{2}a_1 t_1^2 = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{l}{v_x}\right)^2 = \frac{eVl^2}{2mdv_x^2}$$

*Once the electron is past the parallel plates, it no longer feels any force. The electron just slides along with the original* $v_x$ *and the newly acquired* $v_y$*, given by*

$$v_{y\ final} = a_y t_1 = \left(\frac{eE}{m}\right)\left(\frac{l}{v_x}\right) = \frac{eVl}{mdv_x}.$$

*In this force-free region, the electron travels a horizontal distance D at speed* $v_x$*. Therefore, the time spent in this region is simply* $t_2 = \frac{D}{v_x}$*. The vertical displacement during this time is just*

$$y_2 = v_{y\ final}t_2 = \left(\frac{eVl}{mdv_x}\right)\left(\frac{D}{v_x}\right) = \frac{eVlD}{mdv_x^2}.$$

*And so, putting it all together, the total vertical displacement, y, is just,*

$$y = y_1 + y_2 = \frac{eVl^2}{2mdv_x^2} + \frac{eVlD}{mdv_x^2} = \frac{eV}{mdv_x^2}\left(\frac{l^2}{2} + lD\right)$$

*Solving for e/m,*

$$\frac{e}{m}=\frac{yv_x^2d}{Vl\left(\frac{l}{2}+D\right)}$$

5. *Extra credit problem*. Show that **Wien's law**, $u(f,T)=Af^3e^{-Bf/T}$ where A and B are empirical constants, and the **Rayleigh – Jeans law**, $u(f,T)=\frac{8\pi f^2}{c^3}k_bT$, are special cases of the **Planck radiation formula**, $u(f,T)=\frac{8\pi hf^3}{c^3}\frac{1}{(e^{hf/k_bT}-1)}$. What are the empirical A and B in terms of fundamental constants?

*(a) Wien's law is a good approximation for short wavelengths (high frequencies). So we are interested when* $\frac{hf}{k_bT}>>1$ *(high frequency limit). Clearly, in this limit,* $e^{hf/k_bT}$ *is huge. And so,* $\frac{1}{e^{hf/k_bT}-1}\approx\frac{1}{e^{hf/k_bT}}=e^{-hf/k_bT}$. *Therefore,*

$$u(f,T)=\frac{8\pi hf^3}{c^3}\frac{1}{(e^{hf/k_bT}-1)}\approx\frac{8\pi hf^3}{c^3}e^{\frac{-hf}{k_bT}}$$

*which is Wien's law provided that we set* $A\equiv\frac{8\pi h}{c^3}$ *and* $B\equiv\frac{h}{k_b}$.

*(b) Rayleigh – Jeans law is a good approximation for long wavelengths (low frequencies). So, for this case, we are interested when* $\frac{hf}{k_bT}<<1$. *Taylor expanding the exponential for small arguments we find that*

$$e^x=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+...$$

*Keeping the first term in x gives:* $e^{hf/k_bT}\approx1+\frac{hf}{k_bT}$.

*Therefore,*

$$u(f,T)=\frac{8\pi hf^3}{c^3}\frac{1}{(e^{hf/k_bT}-1)}\approx\frac{8\pi hf^3}{c^3}\frac{1}{1+\frac{hf}{k_bT}-1}=\frac{8\pi hf^3}{c^3}\frac{k_bT}{hf}=\frac{8\pi f^2}{c^3}k_bT$$

*which is the Rayleigh – Jeans law.*