PHYS420 (Spring 2002)
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Homework # 4 Solutions

Problems

1. **SMM, Chapter 1, Problem 26.**

When dealing with subatomic particles, it's convenient to express their energy in electron volts (eV) (p.36 SMM). The conversion factor is 1 eV = 1.60 x 10^{-19} J.
The relativistic momentum is given by $p = \gamma mu = \frac{mu}{\sqrt{1-u^2/c^2}}$.
The mass of the proton is $m_{proton} = 1.673\,x 10^{-27}\,kg$.
Useful aid: When converting momentum, which is normally expressed in units of [kg m/s], into [MeV/c] remember that the units of energy [Joule] can also be expressed as [kg $(m/s)^2$] . Of course, c = 3x10^{8} m/s.
Therefore directly plugging in m_{proton} and u…
(a) for u = 0.010 c → p = 5.019 x 10^{-21} kg m/s = 9.41 MeV/c
(b) for u = 0.500 c → p = 2.898 x 10^{-19} kg m/s = 543 MeV/c
(c) for u = 0.900 c → p = 1.036 x 10^{-18} kg m/s = 1943 MeV/c
(d) see (a) – (c)

2. **SMM, Chapter 1, Problem 27.**

(a) Since the electron has a momentum that is 90% larger than the classical momentum, then this means that $p = 1.90\,p_{classical}$. *Therefore,*

$$\frac{mu}{\sqrt{1-u^2/c^2}} = 1.90mu$$

The masses and the top velocity cancel. Solving for u we get,

$$u = c\sqrt{1-\frac{1}{(1.90)^2}} = 0.85c$$

(b) No change. The electron mass canceled out of the equation.

3. **SMM, Chapter 1, Problem 33.**

A proton ($m_{proton} = 1.673\,x 10^{-27}\,kg$*) moves at v = 0.95c.*
(a) $E_{rest} = m_{proton}c^2 = (1.673x10^{-27})(3x10^8)^2 =$ *1.5057x10^{-10} J = 941 MeV.*
(b) $E_{total} = \gamma m_{proton}c^2 = \frac{m_{proton}c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ *=3.20 (941 MeV) = 3013 MeV.*

(c) $E_{Kinetic} = E_{total} - E_{rest}$ *= 3.20 (941 MeV) – (941 MeV) = 2070 MeV.*

4. **SMM, Chapter 1, Problem 36.**

There are various ways of doing this problem (all of them amount to the same thing): ***(1)*** *Since we are given the kinetic energy, we can use* $\{KE = \gamma mc^2 - mc^2\}$ *to find γ and then, the velocity v. This solves part (b) first. The momentum is then given by* $\{p = \gamma mv\}$. ***(2)*** *Alternatively, we can compute the total energy using* $\{E_{total} = \gamma mc^2 = KE + mc^2\}$ *and then use the Energy – Momentum relation* $\{E_{total}^2 = p^2c^2 + (mc^2)^2\}$ *to find the momentum p. We then solve for the velocity using* $\{p = \gamma mv\}$.

I will solve the problem using the first method. A proton has a rest mass energy: $E_{rest} = m_{proton}c^2 = (1.673x10^{-27})(3x10^8)^2 = 1.5057x10^{-10}$ *J = 941 MeV. Therefore, according to* $KE = \gamma mc^2 - mc^2$,

$$(50\ GeV) = \gamma\ (941\ MeV) - (941\ MeV)$$
$$\rightarrow \gamma = 54.135...$$
$$\rightarrow v = 0.999829\ c$$

(a) The momentum is therefore,

$$p_{proton} = \gamma m_{proton} v$$
$$= (54.135)(1.673x10^{-27}\ kg)(0.999829\ c)$$
$$= 2.72x10^{-17}\ kg\ m/s = 50.872\ GeV / c$$

(b) As solved above, v = 0.999829 c.

5. *Follow up to Homework 2, problem 7.* If we solve the scalar relativistic Newton's equation, $F = \frac{\partial p}{\partial t} = \frac{\partial}{\partial t}(\gamma mv)$, where $\gamma \equiv \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$, for the velocity *v(t)*, subject to the conditions of a constant force F_0 and initial velocity v_0, we get:

$$v(t) = \left(\frac{F_0}{m}t + \frac{v_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}\right)\left(1 + \frac{1}{c^2}\left(\frac{F_0}{m}t + \frac{v_0}{\sqrt{1 - \frac{v_0^2}{c^2}}}\right)^2\right)^{-\frac{1}{2}}$$

Consider a small 1 kg object starting from rest, subject to a force F_0 of 100 N.

(a) Using the *non-relativistic* speed equation (i.e. $v(t) = \frac{F_0}{m}t + v_0$), how long would it take for the object to reach the speed of light? Give your answer in days.

(b) Using the *relativistic* speed equation, what's the speed after the same period of time?

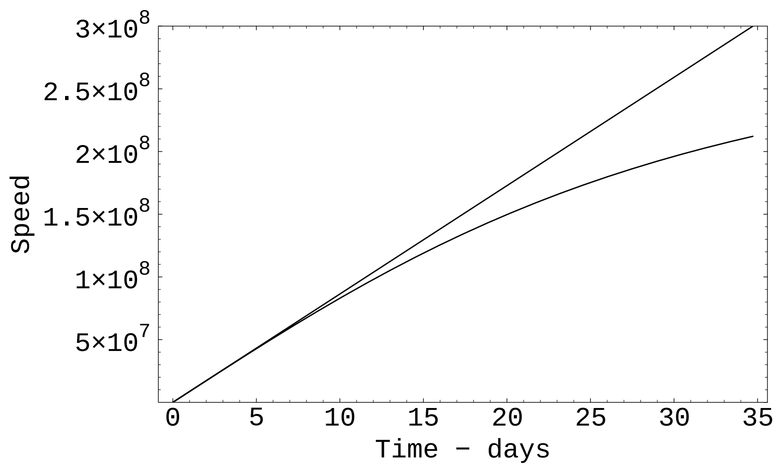
(c) Plot both equations as a function of time from 0 to the time computed in (a)?

Solution:

(a) *Using* $v(t)=\frac{F_0}{m}t+v_0$ *and solving for t we get:* $t=\frac{m}{F_0}\left(v(t)-v_0\right)$. *Plugging in* F_0 *= 100 N, m = 1 kg,* v_0 *= 0 m/s and v(t) = c, we get t = 3x10* ^{6} *sec =**34.7 days.***

(b) Using the relativistic equation and plugging t = 3x10^6 *sec we get v(t) =* ***2.12x10***^{8} ***m/s.***

(c)



6. *Extra Credit Problem 1.* Derive the relativistic speed equation from problem 5. *This looks a lot harder that it is. Give it a try.*

This is a very similar problem to Homework 2, problem 7. The only difference is that we are not looking for x(t) but for v(t). The equations are more complicated but there are less steps. We start with $F_0=\frac{\partial}{\partial t}(\gamma m v)$ *where* $\gamma\equiv\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ *and integrate both sides with respect to time.*

$$\int\frac{F_0}{m}dt=\int\frac{d}{dt}\left(\frac{v(t)}{\sqrt{1-\frac{v^2(t)}{c^2}}}\right)dt$$

$$\frac{F_0}{m}t+c1=\frac{v(t)}{\sqrt{1-\frac{v^2(t)}{c^2}}}$$

Letting t = 0 and solving for c1 we find $c1=\frac{v_0}{\sqrt{1-\frac{{v_0}^2}{c^2}}}$.

Substituting c1 into the original equation,

$$\frac{F_0}{m}t+\frac{v_0}{\sqrt{1-\frac{v_0^{\,2}}{c^2}}}=\frac{v(t)}{\sqrt{1-\frac{v^2(t)}{c^2}}}$$

and solving for v(t) (after a bit of painful algebra) we find,

$$v(t)=\left(\frac{F_0}{m}t+\frac{v_0}{\sqrt{1-\frac{v_0^{\,2}}{c^2}}}\right)\left(1+\frac{1}{c^2}\left(\frac{F_0}{m}t+\frac{v_0}{\sqrt{1-\frac{v_0^{\,2}}{c^2}}}\right)^2\right)^{-\frac{1}{2}}$$

7. *Extra Credit Problem 2*. Show that the relativistic speed equation reduces to the non-relativistic speed equation when *v(t) « c*. *Hint: To do this, we must assume that v0 « c and that t is very small. Mathematically speaking, this is the equivalent of throwing out terms that have v_0^2/c^2 and t^2 in them.*

   We begin the solution by throwing out the obvious v_0^2/c^2 terms directly. This is equivalent to expanding those terms using the binomial theorem and then throwing out the v_0^2/c^2 terms. We are left with,

$$v(t)\cong\left(\frac{F_0}{m}t+v_0\right)\left(1+\frac{1}{c^2}\left(\frac{F_0}{m}t+v_0\right)^2\right)^{-\frac{1}{2}}$$

   Now let's expand the squared term,

$$v(t)\cong\left(\frac{F_0}{m}t+v_0\right)\left(1+\left(\frac{F_0t}{mc}\right)^2+\frac{2F_0v_0t}{mc^2}+\frac{v_0^{\,2}}{c^2}\right)^{-\frac{1}{2}}$$

   Throwing out the t^2 and v_0^2/c^2 terms

$$v(t)\cong\left(\frac{F_0}{m}t+v_0\right)\left(1+\frac{2F_0v_0t}{mc^2}\right)^{-\frac{1}{2}}$$

   We use the binomial theorem since we are looking at t very small,

$$v(t)\cong\left(\frac{F_0}{m}t+v_0\right)\left(1-\frac{F_0v_0t}{mc^2}\right)$$

   Expanding out the terms,

$$v(t)\cong\frac{F_0}{m}t-\left(\frac{F_0t}{m}\right)^2\left(\frac{v_0}{c^2}\right)+v_0-\left(\frac{F_0t}{m}\right)\left(\frac{v_0^{\,2}}{c^2}\right)$$

   And throwing out the t^2 and v_0^2/c^2 terms one last time, we get the desired result:

$$v(t)\cong\frac{F_0}{m}t+v_0$$