**PHYS420 (Spring 2002)**
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**Homework # 1 Solutions**

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## Problems

1. Sign the class list on the instructor's office door. If you are not on the list, add yourself in (print your name clearly) and sign it.

2. Calculate your class identification number (ClassID#) from your social security number (or your UMCP ID number) using the following formula: *ClassID# = SS# mod 999 ,* where x mod(y) gives the remainder of x/y. For example, 999-21-3564 mod (999) will give a remainder of 777 (i.e. $\frac{999213564}{999} = 1000213\frac{777}{999}$ ).

3. For verification purposes, email the instructor your ClassID#. This is also to verify your email address (as listed in the official roster). If you have more than one email, indicate a preference for class communications. Also indicate how often you check your email.

4. A college friend of yours (not science inclined but math literate) asks you to explain to them what a 'Dirac delta function' is. In your own words and taking into account that your college friend has some math knowledge, how would you explain it to them?

   *The* ***Dirac delta function*** *(called the 'delta function' by mathematicians) is a strange beast which, strictly speaking, is not even a function. It is typically written as* $\delta(x)$ *and defined informally as follows:*

$$\delta(x) = \begin{Bmatrix} 0, & if \;\; x \neq 0 \\ \infty, & if \;\; x = 0 \end{Bmatrix}, \quad with \int_{-\infty}^{+\infty} \delta(x)\, dx = 1.$$

   *One can think of it as an infinitely high, infinitesimally narrow spike at the origin, whose area is 1. Technically, it's not a function at all, since it is not finite at x = 0 (mathematicians call it a generalized function or distribution). Nevertheless, it is an extremely useful construct in theoretical physics. Notice that* $\delta(x-a)$ *would be a spike of area 1 at the point a. If you multiply* $\delta(x-a)$ *by an ordinary function f(x), it's the same as multiplying by f(a) because the product is zero everywhere except at the point a. In particular,*

$$\int_{-\infty}^{+\infty} f(x)\delta(x-a)\, dx = f(a)\int_{-\infty}^{+\infty} \delta(x-a)\, dx = f(a)$$

   *That's the most important property of the delta function: Under the integral sign it serves to "pick out" the value of f(x) at the point a. We will see the advantage of this property later in the semester.*

5. A measure of the force $F$ exerted by a stretched spring for small displacements $x$ is given by Hooke's Law $F = kx$. What are the dimensions of the spring constant $k$? Stretching the spring further, the force becomes nonlinear and may be represented by including an anharmonic term, $F = kx + \beta x^3$. What are the dimensions of $\beta$? *(Adapted from a problem by Jeff Simpson)*

*Solving for k we see that k=F/x. By dimensional analysis this means that k has dimensions of* ***[Newtons]/[Meters]****. This has to be so for the product kx to have units of force [Newtons]. Using a similar argument, the anharmonic term ($\beta x^3$) must also have units of force. For this to be so, β needs to have units of* ***[Newtons]/[Meters]$^3$****.*

6. Derive the following expression: $(1+x)^n \cong 1 + nx$ for $|x| << 1$. Explain all your steps. Assume nothing on the part of the grader!

*You will see this expression, known as the Binomial theorem, over and over again throughout the semester. It is worthwhile noting where it came from…*
*This problem deals with the expansion of a function,* $f(x) = (1+x)^n$, into an infinite power series *(i.e. $f(x)=a_0+a_1x+a_2x^2+a_3x^3+\ldots$) where the coefficients of the successive terms of the series ($a_0$, $a_1$, $a_2$, $a_3$, …) involve the successive derivatives of the function. This type of expansion is known as Taylor's expansion and can be written as:*

$$f(x) = \sum_{i=0}^{\infty} \frac{(x-a)^i}{i!} f^{(i)}(a) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \ldots$$

*In our particular example we want to expand our function for very small x; in other words, around x=0. So computing the first couple of terms, we get…*

$$(1+x)^n = (1+0)^n + (x-0)n(1+0)^{n-1} + \frac{(x-0)^2}{2*1} n(n-1)(1+x)^{n-2} + \ldots$$

$$= 1 + nx + \frac{1}{2} n(n-1)x^2 + \ldots$$

*Keeping just the first term…*

$$(1+x)^n \cong 1 + nx$$

7. Estimate the number of piano tuners in New York city? Clearly state all assumptions and approximations used.

*The following is a sample solution illustrating the types of assumptions and estimations that may be done. How might one figure out such a thing? Surely the number of piano tuners in some way depends on the number of pianos. The number of pianos must connect in some way to the number of people in the area.*
***Approximately how many people are in New York City?*** *10,000,000*
***Does every individual own a piano?*** *No*
***Would it be reasonable to assert that 'individuals don't tend to own pianos; families do'?*** *Yes.*

***About how many families are there in a city of 10 million people?***
*Perhaps there are 2,000,000 families in NYC.*
***Does every family own a piano?****No. Perhaps one out of every five does. That would mean there are about 400,000 pianos in NYC.*
***How many piano tuners are needed for 400,000 pianos?*** *Some people never get around to tuning their piano; some people tune their piano every month. If we assume that "on the average" every piano gets tuned once a year, then there are 400,000 "piano tunings" every year.*
***How many piano tunings can one piano tuner do?*** *Let's assume that the average piano tuner can tune four pianos a day. Also assume that there are 200 working days per year. That means that every tuner can tune about 800 pianos per year.*
***How many piano tuners are needed in NYC?***
*The number of tuners is approximately 400,000/800 or* ***500 piano tuners****.*

*Even when using different assumptions for the various factors, it's unlikely that you can justify an answer greater than a factor of 10 or smaller than a factor of 10 from the number originally obtained; that is to say, there are probably not more than 5000 tuners and surely no less than 50. Thus the answer obtained is good to within an "order of magnitude".*