PHYS420 (Spring 2002) **CLASS ID_______________**
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Exam #3
(Monday, May 13th, 2002)

Exam 3 – Quantum Mechanics Formalism

Write your class ID number (NOT your name) on this exam.

Grading breakdown:

Grading breakdown:	**Problem 1**	**3 points**
	Problem 2	**4 points**
	Problem 3	**3 points**
	Problem 4	**10 points**
	Problem 5	**5 points**
	Problem 6	**8 points**
	Problem 7	**5 points**
	Problem 8	**3 points**
	Problem 9	**9 points**
	Problem 10	**14 points**
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	Total points	**64 points**

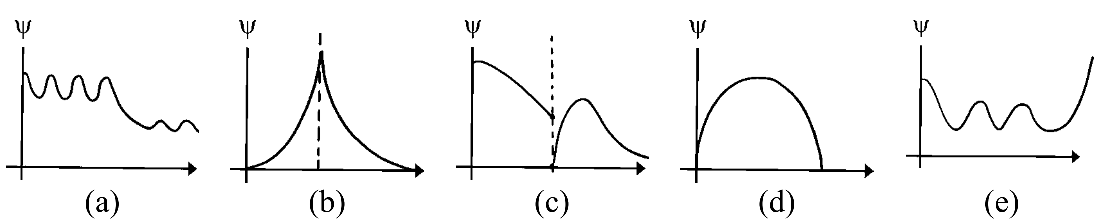
Problems

1. **(3 points)** What is the *physical meaning* of the normalization condition?

2. **(4 points)** Name two *observables* of a particle and state whether they are *sharp* or *fuzzy*.

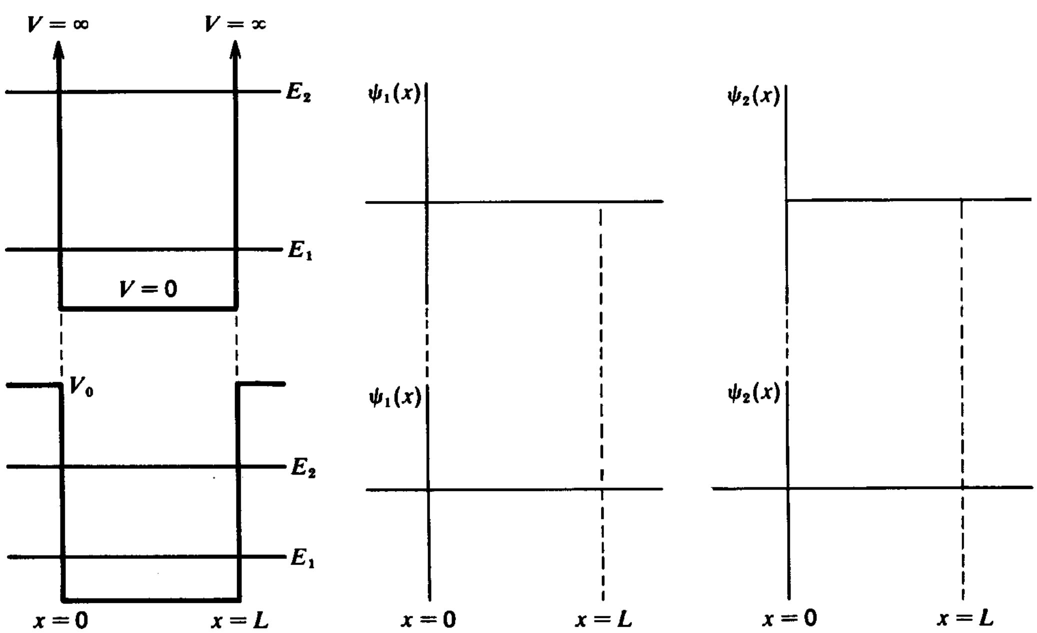
3. **(3 points)** Explain the difference between *phase* and *group* velocities for a wave packet.

4. **(10 points; 2 each)** Of the following functions, which are candidates for the Schrödinger wave function of an actual physical system? For those that are not, state why they fail to qualify.



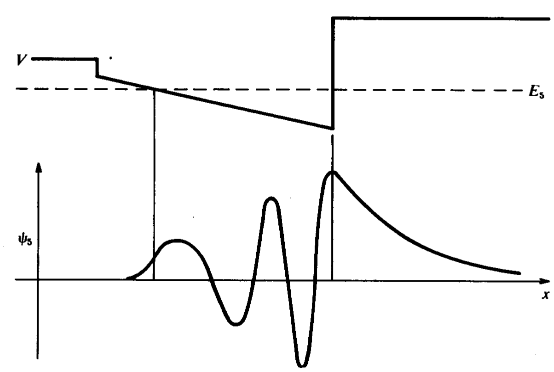
5. **(5 points)** Consider a particle in a box (a.k.a. the infinite square well). Calculate the probability that a particle, in the ground state, will be found in the middle half of the well (i.e. $[\frac{L}{4}, \frac{3L}{4}]$).

6. **(8 points; 2 each)** Graphically compare the 1[st] and 2[nd] bound state of the infinite and finite square well by *carefully* sketching their wave functions in the space provided.

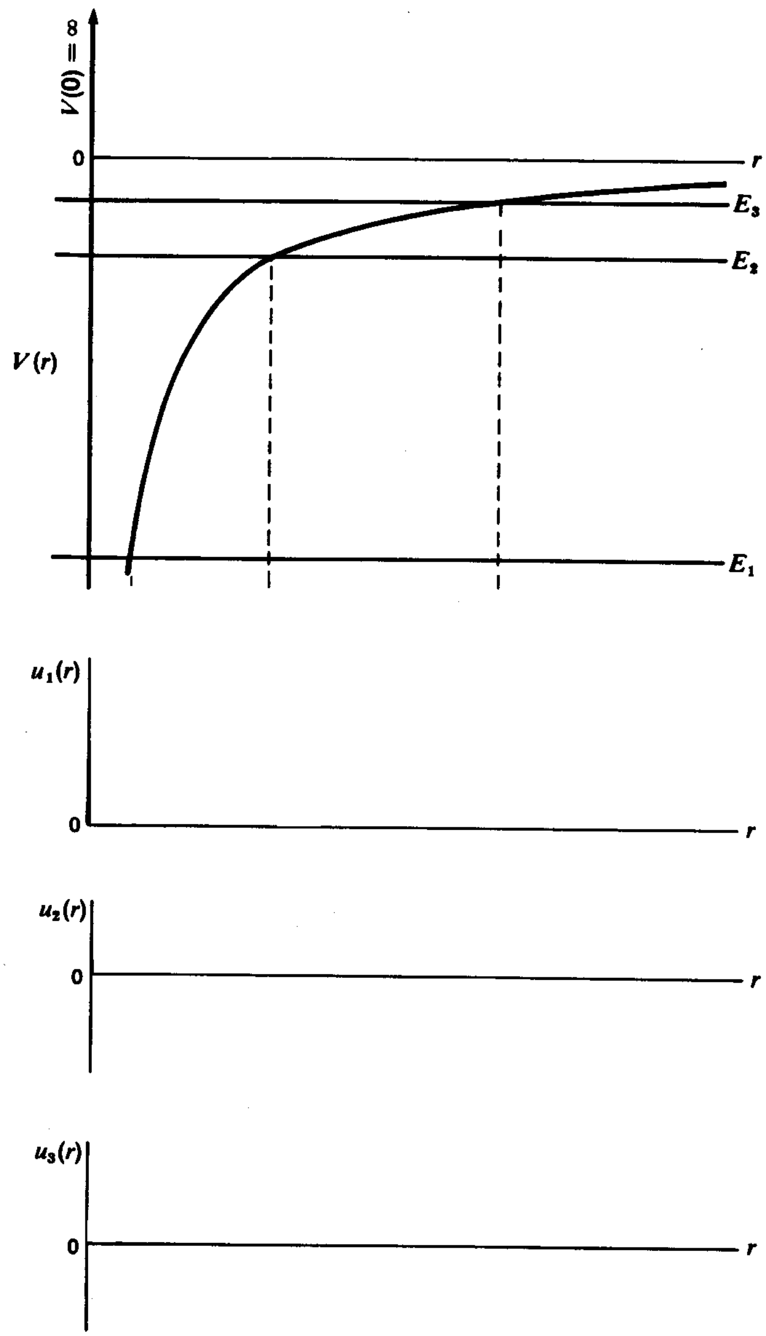


7. **(5 points)** Consider an electron bound in a harmonic oscillator, $E_{total} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$, with an arbitrary uncertainty a ($\Delta x \sim a$). Using the uncertainty principle, (a) *estimate* the value of a that minimizes the total energy and (b) find this minimum energy and compare it to the exact harmonic oscillator ground state energy.

8. **(3 points)** The curve in the figure is alleged to be the plot of a computer-calculated wave function for the fifth energy level of a particle in the diagrammed one-dimensional potential well. Indicate the way in which the plot *fails* to be qualitatively correct.



9. **(9 points; 3 each)** Sketch careful, qualitatively accurate plots (on the space provided) for the 1st, 2nd, and 3rd bound state of a 1-D Coulomb potential. In your plots, don't forget to clearly mark the location of the classical turning points. Notice that $V(r \leq 0) = \infty$. *Important: Check that your wave function has the correct symmetry, number of nodes, relative wavelengths, maximum values of amplitudes and relative rate of decrease outside the well.*



10. **(14 points)** The 2nd bound state (n = 1) wave function of the harmonic oscillator is given by

$$\psi_1(x) = \left(\frac{1}{2a\sqrt{\pi}}\right)^{1/2} 2\left(\frac{x}{a}\right) e^{-\frac{x^2}{2a^2}} \text{ where } a = \sqrt{\frac{\hbar}{m\omega}}.$$

(a) **(2 points)** *Carefully* sketch a qualitative but accurate plot of $\psi_1(x)$. Clearly mark the location of the classical turning points.
(b) **(3 points)** Show that the wave function is normalized.
(c) **(6 points)** Compute the quantum uncertainty in the position, Δx, in terms of *a*.
(d) **(3 points)** What is $< H >$?

EXAM 3 EQUATION SHEET

Wave formalism $\omega = 2\pi f$ $\quad k = \frac{2\pi}{\lambda}$ $\quad v_{phase} = \frac{\omega}{k}$ $\quad v_{group} = \left.\frac{d\omega}{dk}\right|_{k_0}$

$v_{group} = \left.v_{phase}\right|_{k_0} + k\left.\frac{dv_{phase}}{dk}\right|_{k_0}$ $\quad \Delta x \Delta k \approx 1$ $\quad \Delta t \Delta \omega \approx 1$

Matter waves $E = hf$ $\quad \lambda = \frac{h}{p}$ $\quad v_{phase} = c\sqrt{1+\left(\frac{mc}{\hbar k}\right)^2}$

Uncertainty principle $\Delta x \Delta p_x \geq \frac{\hbar}{2}$ $\quad \Delta E \Delta t \geq \frac{\hbar}{2}$

Schrödinger equation $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$

Probabilities & Normalization $P_{[a,b]} = \int_a^b |\psi(x,t)|^2 dx$ $\quad \int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$

Particle in a box $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ $\quad \psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$ $\quad n = 1, 2, 3...$

Harmonic oscillator $[H_{oscillator}] = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2$

$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ $\quad n = 0, 1, 2,...$ $\quad \psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$ *(ground state)*

Expectation values $<Q> = \int_{-\infty}^{\infty} \psi^*(x) Q \psi(x)\, dx$ $\quad \Delta Q = \sqrt{<Q^2> - <Q>^2}$

Operators $<x> = x$ $\quad <p> = \frac{\hbar}{i}\frac{\partial}{\partial x}$ $\quad <H> = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)$

Integrals and Identities

$\int_0^{\infty} u^2 e^{-\beta u^2} du = \frac{1}{4\beta}\sqrt{\frac{\pi}{\beta}}$ $\quad \int_0^{\infty} u^3 e^{-\beta u^2} du = \frac{1}{2\beta^2}$ $\quad \int_0^{\infty} u^4 e^{-\beta u^2} du = \frac{3}{8\beta^2}\sqrt{\frac{\pi}{\beta}}$

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ $\quad \cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$