PHYS420 (Spring 2002)
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Exam #3 Equation Sheet

Wave formalism

$$\omega = 2\pi f \qquad k = \frac{2\pi}{\lambda} \qquad v_{phase} = \frac{\omega}{k} \qquad v_{group} = \left.\frac{d\omega}{dk}\right|_{k_0}$$

$$v_{group} = \left.v_{phase}\right|_{k_0} + k\left.\frac{dv_{phase}}{dk}\right|_{k_0} \qquad \Delta x \Delta k \approx 1 \qquad \Delta t \Delta \omega \approx 1$$

Matter waves

$$E = hf \qquad \lambda = \frac{h}{p} \qquad v_{phase} = c\sqrt{1+\left(\frac{mc}{\hbar k}\right)^2}$$

Uncertainty principle

$$\Delta x \Delta p_x \geq \frac{\hbar}{2} \qquad \Delta E \Delta t \geq \frac{\hbar}{2}$$

Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

Probabilities & Normalization

$$P_{[a,b]} = \int_a^b |\psi(x,t)|^2 dx \qquad \int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$$

Particle in a box

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad \psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \quad n = 1,2,3...$$

Harmonic oscillator

$$[H_{oscillator}] = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2$$

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega \quad n = 0, 1, 2,... \qquad \psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \quad \textit{(ground state)}$$

Expectation values

$$<Q> = \int_{-\infty}^{\infty} \psi^*(x) Q \psi(x)\, dx \qquad \Delta Q = \sqrt{<Q^2> - <Q>^2}$$

Operators

$$<x> = x \qquad <p> = \frac{\hbar}{i}\frac{\partial}{\partial x} \qquad <H> = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)$$

Integrals and Identities

$$\int_0^{\infty} u^2 e^{-\beta u^2} du = \frac{1}{4\beta}\sqrt{\frac{\pi}{\beta}} \qquad \int_0^{\infty} u^3 e^{-\beta u^2} du = \frac{1}{2\beta^2} \qquad \int_0^{\infty} u^4 e^{-\beta u^2} du = \frac{3}{8\beta^2}\sqrt{\frac{\pi}{\beta}}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$