

Exam 2 – Development of Quantum Mechanics



Do NOT write your name on this exam. Write your class ID number on the top right hand corner of each problem page. Do NOT write your class ID number on this page.

Grading breakdown:

Part 1 18 Multiple choice 36 points

Part 2 10 Short answer 36 points

Total points 72 points

Part 1 Multiple choice problems (2 points each).

Questions 1 & 2 refer to the following equation: ${}_{13}\text{Al}^{27} + {}_2\text{He}^4 \rightarrow {}_{15}\text{P}^{30} + X + \text{energy}$.

1. According to the above nuclear reaction, phosphorous (P) is produced by bombarding aluminum (Al) atoms with what type of atomic particle?

- (A) Neutrons.
- (B) Riqons.
- (C) Alpha particles.
- (D) Protons.
- (E) Electrons.

Answer: (C) This question requires you to be able to recognize the symbol for an alpha particle, ${}_2\text{He}^4$. Note that since ion charges are typically not listed in nuclear reactions, this is the same symbol as a helium atom although it's representing a helium nucleus. To be more accurate and complete, we should really write the bare He nucleus as $({}_2\text{He}^4)^{2+}$.

2. According to the above nuclear equation, particle X is which atomic particle?

- (A) Neutrons.
- (B) Riqons.
- (C) Alpha particles.
- (D) Protons.
- (E) Electrons.

Answer: (A) The left side of the equation has a total of 15 protons (13 from Al and 2 from He) and 31 nucleons (21 from Al and 4 from He) which must be accounted for on the right side. Since phosphorous has 30 nucleons and 15 protons and the energy term doesn't carry away any particles, X must have 1 nucleon and no protons. In other words, X must be a neutron. The reaction looks like: ${}_{13}\text{Al}^{27} + {}_2\text{He}^4 \rightarrow {}_{15}\text{P}^{30} + {}_0n^1 + \text{energy}$.

Questions 3 - 5 refer to the following list of names.

- I. J.J. Thompson
- II. H.G.J. Moseley
- III. Ernest Rutherford
- IV. Marie Curie
- V. Niels Bohr

3. This scientist 'discovered' the electron.

- (A) I
- (B) II
- (C) III
- (D) IV
- (E) V

Answer: (A) J.J. Thompson is credited with 'discovering' the electron while performing his famous cathode ray tube experiments.

4. This scientist's model of the atom contained electrons in a uniform, homogenous background of positive charge.

- (A) I
- (B) II
- (C) III
- (D) IV
- (E) V

Answer: (A) J.J. Thompson, after 'discovering' the electron, described the structure of the atom using his 'Plum-pudding' model as being electrons immersed in a background of positive charge like 'raisins in plum-pudding.'

5. This scientist theorized that electrons could only be located in 'quantized' energy levels.

- (A) I
- (B) II
- (C) III
- (D) IV
- (E) V

Answer: (E) The quantum model of the atom is credited to Danish physicist Niels Bohr.

6. If a particular metal in a photocell releases a current when blue light shines on it, it must also release a current when it is struck with

- (A) Ultraviolet light.
- (B) Infrared light.
- (C) Microwaves.
- (D) Radio waves.
- (E) Red light.

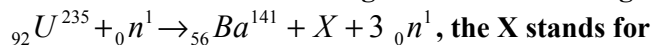
Answer: (A) The photoelectric effect states that electrons will not be ejected from an atom unless incident electromagnetic radiation has at least a minimum frequency value. The only EM radiation having a greater frequency than blue light is ultraviolet.

7. A particle was ejected from the nucleus of an atom in a radioactive decay and the atomic number of the atom increased. The particle was probably

- (A) a hydrogen nuclei
- (B) a neutron
- (C) a proton
- (D) an alpha particle
- (E) a beta particle

Answer: (E) When a beta is emitted from a nucleus, the number of neutrons decreases by one and the number of protons increases by one thereby increasing its atomic number by one but not appreciably changing its mass.

8. When uranium 235 undergoes fission according to the following reaction



- (A) ${}_{51}\text{Sb}^{92}$
- (B) ${}_{51}\text{Sb}^{43}$
- (C) ${}_{36}\text{Kr}^{90}$
- (D) ${}_{36}\text{Kr}^{92}$
- (E) ${}_{55}\text{Ba}^{145}$

Answer: (D) Conservation of positive charge (atomic number, Z) and total number of nucleons (atomic weight, A) requires that X have Z = 36 and A = 92.

9. The laws of photoelectric emission

- (A) are explained by Maxwell's theory of light.
- (B) state that emission is inversely proportional to the intensity of the incident light.
- (C) state that increasing the intensity of the incident light increases the kinetic energy of the photoelectrons.
- (D) state that increasing the frequency of the incident light increases the kinetic energy of the photoelectrons.
- (E) state that the maximum energy to release the electron from a surface is the work function.

Answer: (D) Maxwell's theory of light (the wave theory) does not accurately predict the experimental facts of the photoelectric effect so (A) is incorrect. The emission of photoelectrons is directly proportional to the intensity of the incident light, so (B) is incorrect. The KE is independent of the intensity so (C) is incorrect. Within the range of effective frequencies the KE varies directly with the frequency of the incident light, so (D) is correct. The work function is the minimum energy needed to release the electron so (E) is false.

10. A blackbody radiator

- (A) does not absorb thermal radiation.
- (B) is a perfectly reflecting surface.
- (C) is used in newer automobile engines.
- (D) emits radiation only in the visible light region.
- (E) none of the above.

Answer: (E) None of the above. A blackbody DOES absorb thermal radiation (A is incorrect), DOES NOT reflect it (B is incorrect), and emits well beyond the visible range (D is incorrect). Of course C is just silly.

11. A nuclide with a half-life of 2 days is tested after 6 days. What fraction of the sample *has decayed*?

- (A) 1/8
- (B) 1/4
- (C) 1/2
- (D) 3/4
- (E) 7/8

Answer: (E) After 6 days, we have gone through 3 half lives. In other words, 1/2 of 1/2 of 1/2 of the sample (i.e. 1/8) remains. Conversely, 7/8 has decayed.

12. Several classical models were proposed to explain the observed blackbody's spectral emittance. In one particular derivation it was found that it diverged at short wavelengths. This result is known as

- (A) Stefan's law.
- (B) the ultraviolet catastrophe.
- (C) the Compton effect.
- (D) Wien's law.

Answer: (B) One of the most dramatic failures of the Rayleigh – Jeans classical calculation for blackbody radiation is that the spectral emittance depends on f^2 ; which obviously diverges at high frequencies (short wavelengths).

13. In the Compton experiment, the wavelength of the scattered light is _____ the wavelength of the incident light.

- (A) longer than
- (B) the same as
- (C) shorter than

Answer: (A) According to $\lambda' = \lambda_0 + \frac{h}{m_e c}(1 - \cos \theta)$, we see that the wavelength of the scattered light gets longer.

14. Which quantity(ies) is(are) quantized in the Bohr atom?

- (A) the electron orbit.
- (B) the electron energy.
- (C) the electron angular momentum.
- (D) all of the above.
- (E) two of the above.

Answer: (D) The electron's angular momentum is explicitly quantized in Bohr's fourth postulate. As a consequence of this, the electron's orbit and energy are also quantized.

15. In the Bohr atom, the laws of classical mechanics apply to

- (A) the orbital motion of the electron in a stationary state.
- (B) the motion of the electron during transitions between stationary states.
- (C) both of the above.
- (D) neither of the above.

Answer: (A) According to Bohr's first postulate, the orbital motion of the electron is circular and given by the classical Coulomb force.

16. A beam of ultraviolet light is incident on a metal disk electrically isolated from the environment. Which statement(s) is(are) true?

- (A) If the metal disk was initially positively charged, it discharges.
- (B) If the metal disk was initially negatively charged, it discharges.
- (C) Both of the above.
- (D) Neither of the above.

Answer: (B) When the ultraviolet radiation strikes the metal disk, photoelectrons are emitted from the metal. Thus a metal disk, which is initially negatively charged, will discharge. If the metal disk is initially positively charged, any electron that is emitted is immediately attracted back to the disk. The same happens if the disk is initially neutral because each emitted photoelectron leaves a positive charge behind in the disk.

17. A xenon arc lamp is covered with an interference filter that only transmits light of 400 nm wavelength. When the transmitted light strikes a metal surface, a stream of electrons emerges from the metal. If the intensity of the light striking the surface is doubled,

- (A) more electrons are emitted in a given time interval.
- (B) the electrons that are emitted are more energetic.
- (C) both of the above.
- (D) neither of the above.

Answer: (A) The increased intensity of the light source will increase the number of photoelectrons emitted per unit time. Since the frequency, and hence the photon energy, of the additional light source is equal to that of the original source, the energy of the ejected electrons will not change.

18. In order for an atom to emit light, it must first be ionized. This statement is

- (A) true
- (B) false

Answer: (B) For an atom to emit light, one of its electrons must release energy by making a transition from a higher orbit to a lower one. Ionizing an atom just means that the electron has received enough energy to escape the atom, which, by itself, does not result in emission of light.

Part 2 Short answer and computational problems.

1. (2 points) State two experimental features of line emission spectra.

Answer:

1. The spectrum is not continuous; it consists of a few bright lines of color.
2. The emission is highly element specific.

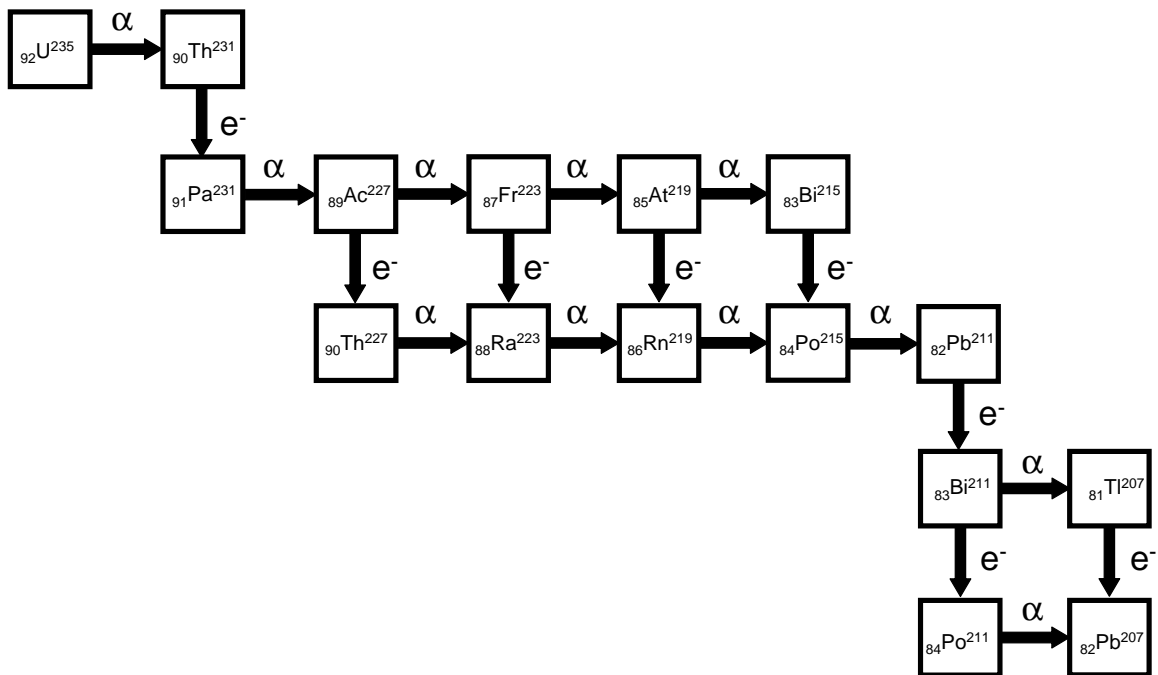
2. (2 points) State two experimental features of blackbody emission spectra.

Answer: You can write any two of the following.

1. The spectrum is continuous.
2. The light distribution is independent of the composition of the blackbody.
3. The total radiated power varies as temperature to the fourth power.
4. The peak of the curve shifts with temperature.

3. (4 points) In the Actinium series, ${}_{92}\text{U}^{235}$ naturally decays into the stable isotope ${}_{82}\text{Pb}^{207}$.

Enter the correct isotope symbol in each open square.



4. (3 points) Consider an ideal blackbody at T_0 . Increasing the temperature by a factor of 2 does what to the frequency of the peak emission? Show all of steps.

Answer: According to Wien's displacement law, $\lambda_{peak}T = const$. If I increase the temperature by a factor of 2 (i.e. $T_1 = 2T_0$) then the frequency increases by a factor of 2.

Proof: $\lambda_0T_0 = const = \lambda_1T_1$

$$\lambda_0T_0 = \lambda_1(2T_0)$$

$$\lambda_1 = \frac{\lambda_0}{2}$$

$$\frac{c}{f_1} = \frac{c}{2f_0}$$

$$f_1 = 2f_0$$

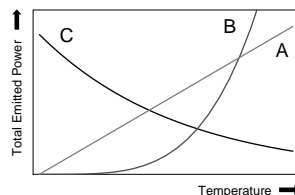
5. (3 points) Supposed that we have a Carbon atom ($Z = 6$) that has had all of its orbiting electrons removed except one. Do you expect the first Bohr orbit to be larger or smaller than it was in a hydrogen atom? By what factor? Explain physically why you would expect this to be so?

Answer: The remaining electron acts like it would around a hydrogen atom, except that the attractive force from the nucleus is six times larger than it would be in hydrogen. Therefore we expect the electron to be bounded in a much smaller orbit. According to the

predictions of the Bohr atom, $r_n = \frac{n^2 a_0}{Z}$, the first Bohr orbit for Carbon is six times

smaller in radius (i.e. $r_{1Carbon} = \frac{a_0}{6} = \frac{1}{6}r_{1Hydrogen}$).

6. (3 points) Consider the following schematic plot of the *total* power per unit area, emitted at all frequencies, as a function of temperature. Which one of these three curves corresponds to a blackbody? Explain your reasoning. *Both axes are linear scales.*



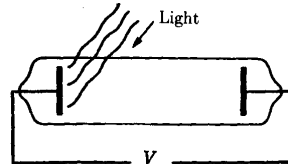
Answer: (B) For a true blackbody, the total emitted power is given by Stefan's law and is proportional to the temperature to the fourth power. Using the fact that the curves are plotted on linear scales, we can see that curve A is linear, which is the incorrect scaling. Curve C has an inverse dependency (slope is always negative), which is also the incorrect scaling. By process of elimination, Curve B is the only candidate that could

match a blackbody. It also begins at the origin and it's concave up (slope is always positive) making it a strong candidate for a power dependence on T .

7. (3 points) Consider the following set of atoms / ions: H^+ , He^{2+} , Li^{2+} , Be^{5+} , B. To which ones can we apply Bohr's quantized energy formula? Explain your reasoning.

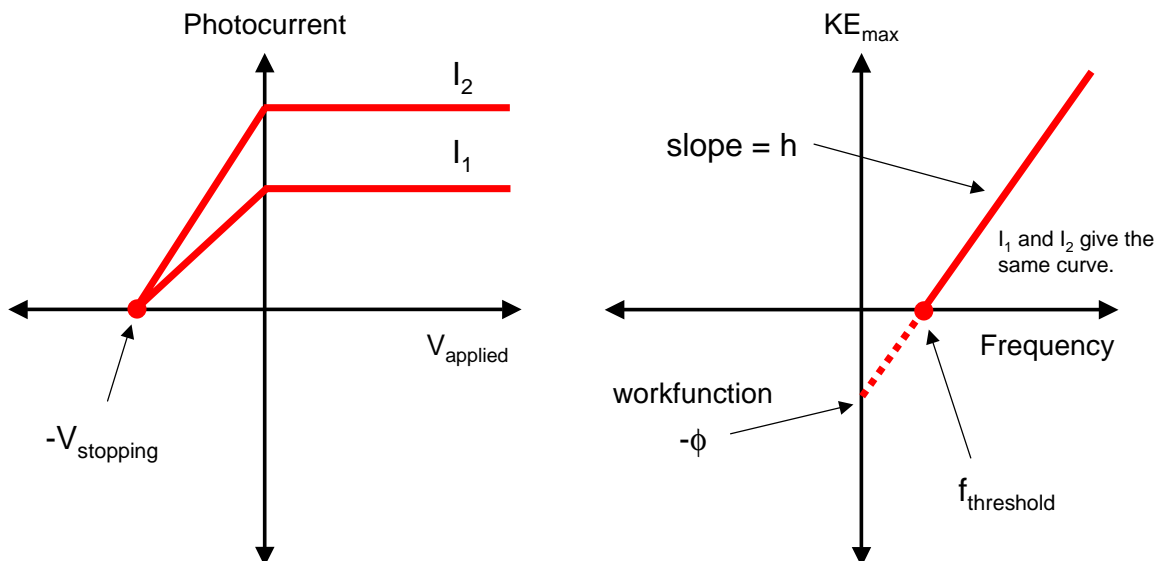
Answer: Only to Li^{2+} . Bohr's theory applied to atoms with a single electron orbiting. This means that we can only apply it to a system that has **all but one** electron removed. Upon inspection of our set of atoms/ions, we see that H^+ has 0 electrons, He^{2+} has 0 electrons, Li^{2+} has 1 electron, Be^{5+} doesn't exist because it only has 4 electrons available and B has all 5 electrons.

8. (8 points) Consider two light beams of different intensities (I_1 and I_2 such that $I_2 > I_1$) incident on the photoelectric circuit shown.

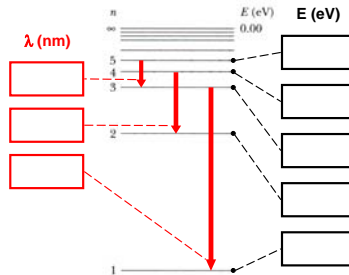


Plot schematically, on the given blank graphs, (1) the photocurrent (i.e. number of ejected electrons per unit time) as a function of applied voltage and (2) the maximum kinetic energy of the photoelectrons as a function of frequency for each beam. Label as many features as you can.

Answer: See page 71-72 of SMM for a full explanation.



9. (4 points) Consider the energy level diagram below to be for hydrogen-like Helium (i.e. He^+). Work out the energy of the given levels (in eV) and the wavelengths of the given transitions (in nm).



Answer: The energy levels for a hydrogen-like ion whose charge number is Z are given by $E_n = -13.6 \text{ eV} \frac{Z^2}{n^2}$. The wavelengths of the transitions are then given by $\lambda = \frac{hc}{\Delta E}$. For helium, ($Z = 2$), we get the following:

$$E_1 = -54.4 \text{ eV}, E_2 = -13.6 \text{ eV}, E_3 = -6.04 \text{ eV}, E_4 = -3.40 \text{ eV}, E_5 = -2.18 \text{ eV}.$$

$$\lambda_{3 \rightarrow 1} = \frac{1239.7 \text{ eV} \cdot \text{nm}}{(-6.04 \text{ eV}) - (-54.4 \text{ eV})} = 25.6 \text{ nm}, \lambda_{4 \rightarrow 2} = 121.5 \text{ nm}, \lambda_{5 \rightarrow 3} = 321.2 \text{ nm}.$$

10. (4 points) State an experimental effect (don't just name experiments) that illustrates the following features of light and matter.

*** Example***

Experimental effect illustrating a prediction of special relativity:

The right way to answer: the speed of light in vacuum is the same in all directions.

The wrong way to answer: the Michelson – Morley experiment.

(A) the wave nature of light.

Either the diffraction of light passing through a small hole or the interference of light after going through two or more slits.

(B) the particle nature of light.

The Compton effect or the photoelectric effect.

(C) the particle nature of an electron.

Electrons have mass and charge and they leave observable tracks in cloud chambers.

(D) the wave nature of an electron.

Diffraction of an electron beam after scattering off a crystal surface.