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PHYSICS 410 | Prof. S. J. Gates
Fall 2005 | Mid-Term Exam | Oct. 27, 2005

This is a OPEN book examination. Read the entire examination before you begin to work. Be sure to read each problem carefully. Any questions should be directed to the proctor. There is an hour & fifty minute time limit. Show all of your work. Use the backs of pages if necessary or request an extra booklet. Be sure to complete the front page of the examination booklet including your name. Show all calculations needed to support your answers, where necessary. Most importantly, THINK before you start to calculate.

Problem (1.)

A prism has four of its vertices located at (0, 0, 0), (a, 0, 0), (0, b, 0) and (a, b, 0) in the x-y plane and four of it vertices located at (0, 0, c), (a, 0, c), (0, b, c) and (a, b, c) above the x-y plane. The prism has a mass of $\mathcal{M}$ and a constant mass density.

(a.) Find the location of the center of mass of the prism.

(b.) Find the explicit form of the moment of inertia tensor, $\mathcal{I}_{i\,j}$.

(c.) Now we choose a non-standard coordinate system in which the moment of inertia tensor takes the form,

$$\mathcal{I}_{i\,j} \;=\; \mathcal{M}\,\{\,a^2 \,+\, b^2 \,+\, c^2\,\} \begin{pmatrix} m_{11} & m_{12} & 0 \\ m_{12} & m_{22} & 0 \\ 0 & 0 & m_{33} \end{pmatrix}$$

where the m's are just constants. Find the principal axes.

Problem (2.)

An electron is launched vertically upward with an initial speed of V_0 into a medium with a resistance described by $\vec{f} = -\,c\,|\vec{v}|\,\vec{v}$. This experiment is conducted in the space shuttle while it is orbit.

(a.) Find the equation that describes the position of the electron as a function of time.

(b.) If we assume the electron is effectively at rest when it reaches a velocity that is 10^{-3} of its initial velocity, to what height does it reach?

Problem (3.)

In orbit inside the space shuttle, a globe of the Earth (a thin spherical shell mass M, radius R) is at rest and centered on the origin. An astronaut throws a tennis ball (mass m and speed V) toward the globe from the position described by $\alpha R\widehat{x}$. The ball strikes the globe at a position described by the vector $\vec{r}_C = \frac{1}{\sqrt{3}} R[\,\widehat{x} + \widehat{y} + \widehat{z}\,]$. After the collision, the ball is observed to move with a velocity given by

$$\vec{v}_f = \Big[\,\frac{V\,(\,\widehat{y} + \widehat{z}\,)}{\sqrt{(1-\alpha)^2 + 2}}\,\Big] \quad .$$

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(a.) Find the velocity of the globe at the instant after the collision.

(b.) Find the angular velocity of the globe at the instant after the collision.

(Hints: Treat the space shuttle as an inertia frame and treat the tennis ball as a point particle.)

Problem (4.)

The earth is actually slowing down in its spinning about its axis. Let us model this by writing

$$\vec{\Omega} \;=\; \Omega_0\,[\,1 \;-\; (\frac{\alpha_0}{\o_0})t\,][\,sin\lambda\,\hat{r} \;-\; cos\lambda\,\hat{\theta}\,] \quad ,$$

where $\Omega_0 = 7.5 \times 10^{-5} s^{-1}$ and $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are the standard unit vectors of a spherical coordinate system.

(a.) A ball is dropped from a height of $\mathcal{H}$ (where $\mathcal{H} << $ the radius of the earth) at co-latitude λ. Relative to the position the ball lands when ignoring the slowing of the earth's spin, give the position where the ball strikes the ground.

Problem (5.)

A force $\vec{F}$ is given by (with f_2 & R being constants)

$$\vec{F} \;=\; \Big[\frac{f_2 \, [\, x^3 \, \widehat{x} \;-\; x \, z^2 \, \widehat{y} \;-\; 2 \, x \, y \, z \, \widehat{z} \,]}{(R)^3} \Big] \quad .$$

Calculate the line integral of this force, on the surface of a sphere of radius R from the point with coordinates $(R,\, 0,\, 0)$ to the point with coordinates $(-\, R,\, 0,\, 0)$ along two *distinct* paths.

(a.) The first path should be along the ‘equator’ of the sphere.

(b.) The second path should be the shortest one that includes the point $\vec{r}_C$ given in problem three.