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PHYSICS 410 — Prof. S. J. Gates
Fall 2005 — Final Exam — Dec. 16, 2005

This is a OPEN book examination. Read the entire examination before you begin to work. Be sure to read each problem carefully. Any questions should be directed to the proctor. There is an hour & fifty minute time limit. Show all of your work. Use the backs of pages if necessary or request an extra booklet. Be sure to complete the front page of the examination booklet including your name. Show all calculations needed to support your answers, where necessary. Most importantly, THINK before you start to calculate.

Problem (1.)

An asteroid is observed to be orbiting around a gaseous cloud in such a way that its angular momentum L and kinetic energy K satisfy the equation $K/L^2 = A_0 exp[2\theta\,]$. Use circular coordinates in the following question.

(a.) Using the inverse of the radial distance, (i.e. $r = u^{-1}$) and θ as the independent variable, derive an expression for $\frac{K}{L^2}$ and use it to find $u(\theta)$.

(b.) If the angular momentum is given by $L_0 exp[-2(t/\tau_0)\,]$ find the potential energy for this problem.

Problem (2.)

Consider two pieces of wire each with mass M_w (with uniform densities). Both are bent into the shape of a semi-circle. One wire 'sits' in the x-y plane and the other in the x-z plane. Both wires have their ends resting on the points (-4, 0) and (4, 0).

(a.) What is the location of the center of mass?

(b.) What is the moment of inertia tensor for the system of wire?

(c.) What is the rotational kinetic energy of the wire system?

Problem (3.)

A Klingon Warbird and the Starship Enterprise are approaching the planet Vulcan but from opposite directions. Each has standard 'running lights' (flashing lights on the exterior). Each captain sends a message to a scientist on the planet that the frequency of their light is 430 trillion Hz. However, using a powerful sensors the scientist observes the following data

Table 1: Sensor Data

	Mass	Frequency	Length
Enterprise	190 million kg	680 trillion Hz	1000 m
Warbird	200 million kg	720 trillion Hz	1250 m

(a.) From the deck of the Enterprise, how fast does the Warbird appear to be approaching?

(b.) From the deck of the Warbird, what would be the observed mass of the Enterprise?

Problem (4.)

Imagine you are in an airplane that is slowly spiraling in for a landing. The position vector of the airplane as described by someone on the ground is given by

$$\vec{R}_p = \rho_0 cos(\omega_0 t)\widehat{x} - \rho_0 sin(\omega_0 t)\widehat{y} + [\, H_0 - v_0 t \,]\widehat{z}$$

(a.) What are the acceleration and velocity vectors of the airplane?

(b.) When the airplane lands, how many complete rotations and fractions thereof has it made?

(c.) Now sitting in your seat facing the front of the plane, you decide to define **mutually** orthogonal unit vectors with you at their origin. The direction when you look straight ahead is your 'x-direction' and the direction of your left shoulder is your 'y-direction.' Finally your 'z-direction' is perpendicular to these two. Write explicit expressions for 'your' unit vectors as described by someone on the ground.

(d.) In your frame of reference, write Newton's Second Law.

Problem (5.)

A bead of mass M is constrained to slide along the frictionless surface of a sphere of radius R_0. There is a potential energy associated with the position of the given by

$$U(\vec{r}) \;=\; M\,A_0\,[\;\ell_1\,x \;+\; \ell_2\,y \;+\; \ell_3\,z\;]$$

(a.) Find the Lagrangian for this system and express it in terms of two angles and their time derivatives.

(b.) Find the equation of motion of this system via the Euler-Lagrange equations.

(c.) Find the Hamiltonian of this system.

Problem (6.)

Now two particles of mass M_1 and M_2 with coordinates $(x_1,\,y_1,\,z_1)$ and $(x_2,\,y_2,\,z_2)$ are constrained to the surface of the same sphere. The potential energy of the new system is given by

$$U_{Total} \;=\; U(\vec{r}_1) \;+\; U(\vec{r}_2) \;+\; \frac{1}{2}k_A\,R^2{}_0\,(6\theta_1 \;-\; 5\theta_2)^2 \;+\; \frac{1}{2}k_B\,R^2{}_0\,(\theta_1)^2 \;+\; \frac{1}{2}k_C\,R^2{}_0\,(\theta_2)^2$$

(a.) What is the form of Newton's second law?

(b.) Solve completely to describe the equation of motion for this system. Include any appropriate discussion of normal modes, eigenmodes and eigenfrequencies.