Phys 410 – Homework #7

All problems from Taylor.

1) 13.5 (3 pts)
2) 13.13 (3 pts)
3) 13.28 (a and b) (6 pts)
4) Read Taylor section 7.9, then do 13.18 part (a) only (3 pts).

   A few comments:

   a. **A** is the vector potential; the magnetic field (**B**) is derived from the vector potential by taking its curl: $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$.
   b. You should derive the Hamiltonian by applying the definition given in Taylor's equation 13.22. The sum will be over the three spatial dimensions (x,y,z).
   c. To get the quantum mechanical Hamiltonian for a charged particle in a magnetic field, we take inspiration from this classical Hamiltonian and substitute $\boldsymbol{p} \to \frac{\hbar}{i}\boldsymbol{\nabla}$ to find:

$$\hat{H} = \frac{1}{2m}\left(\frac{h}{i}\boldsymbol{\nabla} - q\boldsymbol{A}\right)^2 + qV$$

   This turns out to be the correct Hamiltonian operator for this quantum system. See also Griffiths *Introduction to Quantum Mechanics*, 2^{nd} edition, Problems 4.59 to 4.61.

5) Extra Credit (3 pts) A simple plane pendulum consists of a mass on a string of length L. After the pendulum is set in motion, the length of the string is slowly shortened at a constant rate (α). The suspension point remains fixed. Compute the Lagrangian and Hamiltonian functions. Compare the Hamiltonian to the total energy, and discuss the conservation of energy for the case of small oscillations.

Optional problems, for further study. If you attempt one of these, we will read your solution and give you written feedback. No extra credit. Solutions will be posted.

6) 13.11
7) 13.27