Phys 410 – Homework #4

Numbered problems are from Taylor.

1) 5.13 (3 pts)
2) 5.49 & 5.53 (a and b) (9 pts total) Computer problem. 5.53 is a continuation of 5.49.
3) The Green's function for a linear oscillator that starts from rest is

$$G(t,t') = \begin{cases} \frac{1}{m\omega_1} e^{-\beta(t-t')} \sin(\omega_1(t-t')), \text{ for } t \geq t' \\ 0, \text{ for } t < t' \end{cases}$$

The solution for a forced oscillator with a forcing function F(t) is then

$$x(t) = \int_{-\infty}^{t} F(t')G(t,t')dt'$$

a) (3 pts) Calculate x(t) for an oscillator starting from rest for the following case
   - it is undamped ($\beta = 0$);
   - it has natural frequency ω_0;
   - it is driven by the following force function: F = 0 before t = 0, is constant with value $F = F_0$ for $0 < t < \tau$, where $\tau = 2\pi/\omega_0$, and is zero again for $t > \tau$.

b) (3 pts) Make a plot or sketch of the resulting motion of the oscillator.
c) (3 pts) Give an intuitive explanation in terms of work and energy for why the oscillator behaves the way it does for time $t > \tau$.

4) Extra Credit (3 pts) A mass (m) moves in one dimension (x). It experiences a positive constant force ($+F_0$) when x < 0, and a negative constant force ($-F_0$) when x > 0. No other forces are present. Calculate the period of the oscillatory motion, in terms of the amplitude of the motion (A), the mass (m), and the force (F_0). Sketch a plot of velocity (y-axis) versus position (x-axis) for one full period. (This type of plot is called a *phase space diagram*.)

Optional problems, for further study. If you attempt one of these, we will read your solution and give you written feedback. No extra credit. Solutions will be posted.

5) 5.26
6) 5.24
7) 5.41