Phys 410 – Homework #10

All problems from Taylor.

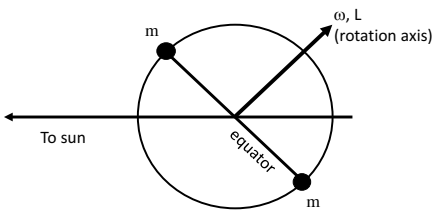
0) (zero pts) Consider watching the youtube videos linked on the course website under the following categories. Most of these videos are very short (one minute or less).

- Torque-free rotations: Tennis Racket theorem ("Dzhanibekov effect" in Russia)
- Precession of a torque-free symmetric top
- Precession of the equinoxes
- Gyroscopes, with gravitational torque.

1) (3 pts) 10.41. Consider the time derivative of $(\omega_1)^2 + (\omega_2)^2$.
2) (3 pts) 10.42. Look up the form of the inertia tensor of a solid rectangular object on the Wikipedia page for "List of moments of inertia".
3) (3 pts) 10.47. You will need the inertia tensor of the earth + mountain system. Choose a coordinate system with the z-axis pointing from the center of the earth toward the mountain's location; then add together the inertia tensor for a solid sphere (earth) and for a point particle (mountain).
4) (9 pts) 10.54 (a, b, & c) (computer problem)

Extra Credit. The earth has excess mass around its equator of magnitude m ~ 10^{-3} $M_{earth.}$ The gravitational field of the sun acts upon this excess mass and causes the earth to experience a torque. This torque, along with a similar torque from the moon, causes the earth's angular momentum and omega vectors to slowly trace out a circle on the sky, a phenomenon known as the precession of the equinoxes.

We can make a simple estimate of the time required for the equinoxes to precess once. Start by estimating the size of the external torque due to the sun. Hint: consider the excess mass to be localized to two point particles, both on the earth's equator. At a particular instant, one mass is on the side closest to the sun, and the other is opposite. See diagram below. How large is the torque? Since this is an estimate, you may neglect angular factors in your estimate, considering these to be near unity, as well as factors of two.



Then compute the precession period for the earth's rotation axis by consider the earth to be similar to a gyroscope. You may neglect numerical factors in the moment of inertia expressions, assuming these to be close to one. You will need to look up some astronomical data to answer this problem. Also look up the accepted value for the precession period and compare to your estimate.

Optional problems, for further study. No extra credit. Solutions will be posted.
1) 10.44. 2) 10.43. 3) 10.56.