Cross Product Literacy

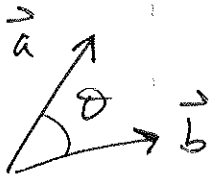
This is a refresher on how to compute cross products, including advice for which method to employ.

$$\vec{c} = \vec{a} \times \vec{b}$$

Method 1

If you are given the angle between the two vectors $\vec{a}$ and $\vec{b}$, and you are given their magnitudes, then compute the magnitude and direction of $\vec{c}$ in a 2 step process

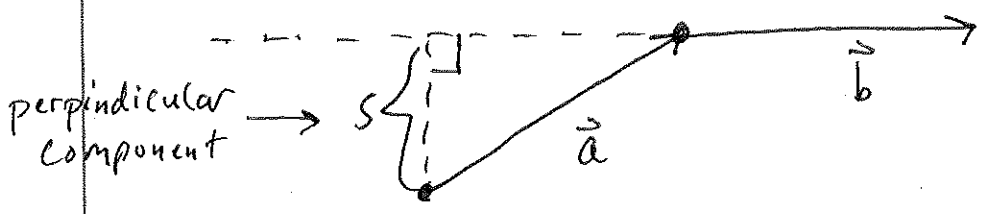
step 1: $|\vec{c}| = |\vec{a}||\vec{b}| \sin\theta$



step 2: use right hand rule to figure out the direction.

In this example, $\vec{a} \times \vec{b}$ is into the page.

Method 2 Sometimes we are given the perpindicular component of vector $\vec{a}$ compared to $\vec{b}$:



Then:

step 1: magnitude of $\vec{c}$ is $|\vec{c}| = s|\vec{b}|$ or sb

step 2: use right hand rule to figure out the direction. In this example, direction is into the page.

Method 3

If we are given the x, y, and z components of both vectors, then we can compute with brute force:

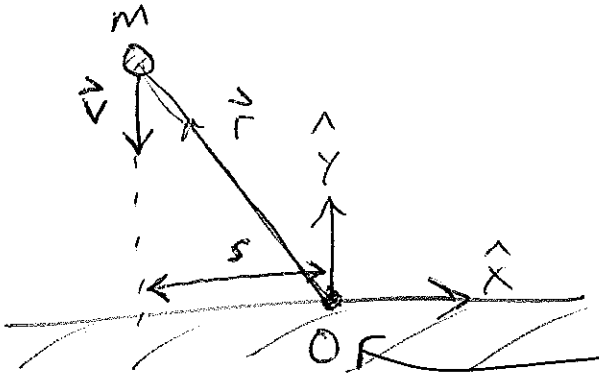
Example: $\vec{a} = 2\hat{x} + 3\hat{y} + 6\hat{z}$

$\vec{b} = 9\hat{y} + 2\hat{z}$

Then

$$\vec{c} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & 3 & 6 \\ 0 & 9 & 2 \end{vmatrix} = (-54)\hat{x} - 4\hat{y} + 18\hat{z}$$

(1) Ball falling with non-zero impact parameter (s)



$$\vec{L} = \vec{r} \times m\vec{v}$$

a) Is $\vec{L}$ zero or non-zero?

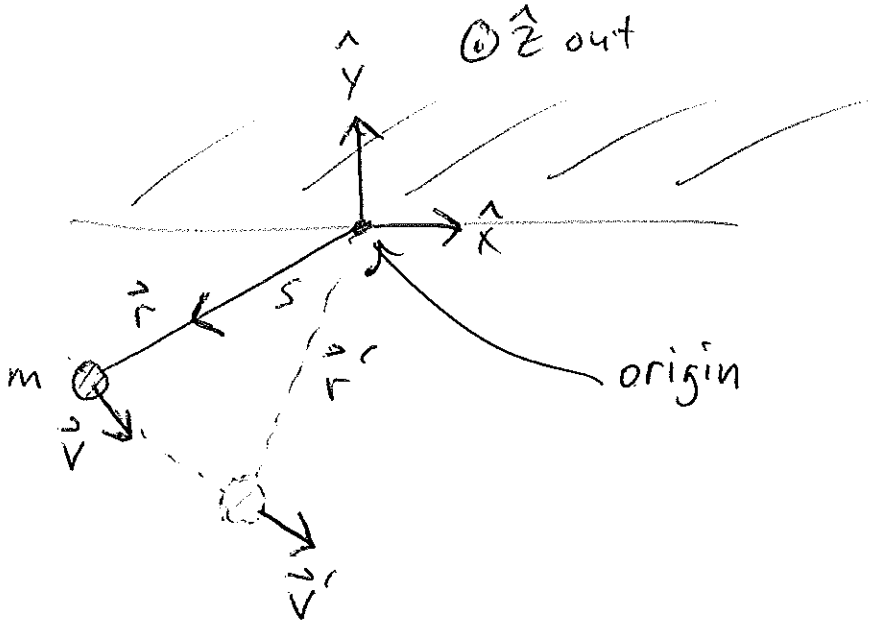
b) If non-zero, what direction?

c) If non-zero, what magnitude?

d) Is $\vec{L}$ changing as the ball drops and accelerates?
If yes, is the magnitude, the direction, or both changing?

(2) Repeat but with the origin directly underneath the ball. → (zero impact parameter.)

(3) Swinging Pendulum



a) Is $\vec{L}$ zero or non-zero

b) If non-zero, what direction?

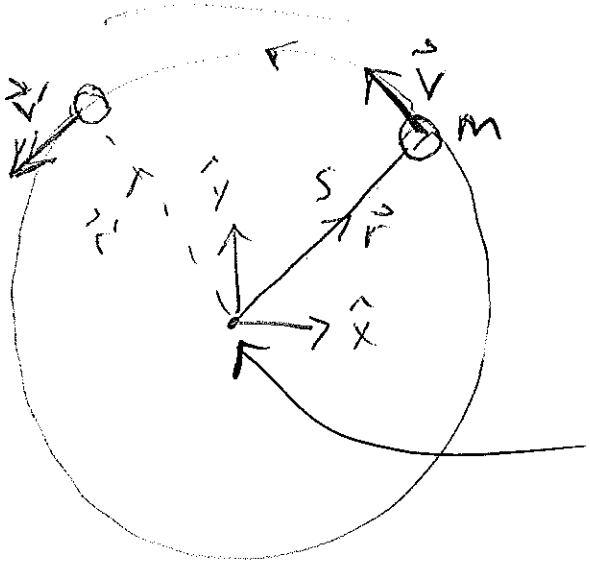
c) If non-zero, what magnitude?

d) Is $\vec{L}$ changing as the pendulum swings?
If yes, is magnitude changing? Is direction changing?

e) Can we pick an origin that makes $\vec{L}$ zero at the moment shown?

f) Can we pick an origin that makes $\vec{L}$ zero for all time?

(4) Circular motion - Tetherball, no gravity, just rope tension.



bird's eye view.

a) Is $\vec{L}$ zero or non-zero at the moment shown?

b) If non zero, what direction?

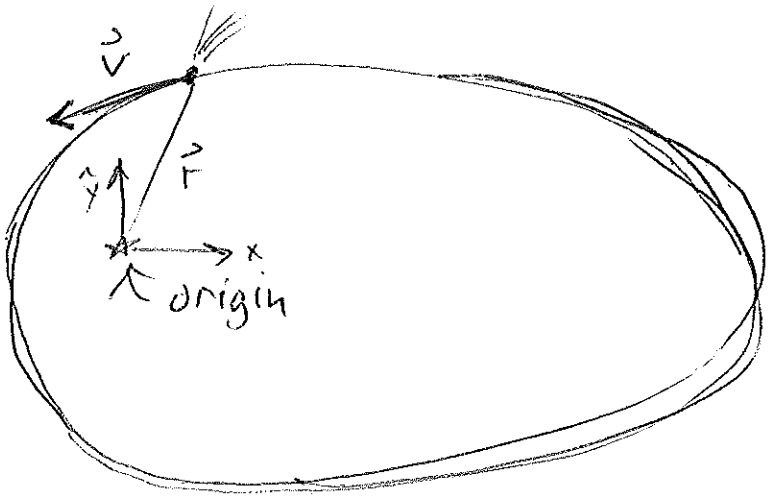
c) If non-zero, what magnitude?

d) Is $\vec{L}$ changing?

If yes is magnitude changing? Is direction changing?

e) Can we pick an origin that makes $\vec{L}$ zero at the moment shown? That makes $\vec{L}$ zero for all time?

⑤ Comet in elliptical orbit



a) Is $\vec{L}$ zero or non-zero?

b) If non-zero, what direction?

c) If non-zero, what magnitude?

d) Is L changing? (Answer is not obvious unless we know the force is central, and that $\vec{L} = \vec{r} \times \vec{F}$).