Physics 404

HOMEWORK ASSIGNMENT #10

Fall 2014

The last, at last!!

## The third quiz will be on Tuesday, Dec. 9 (one lecture later than in the original syllabus).

**Due date:** Thursday, Dec. 4 **Deadline:** Tuesday, Dec. 9

- 1. (7) 7.61 Low-temperature heat capacity of liquid <sup>4</sup>He. You do not need to repeat the derivation for phonons; just cite the relevant results and where they are modified here. It is convenient to consider one mole of <sup>4</sup>He, i.e.  $N = N_A$ . Note that we do not use the fact that <sup>4</sup>He becomes a superfluid below about 2 K.
- 2. (12) 7.64 Generalization of lattice vibrations to spin waves (and so phonons to magnons) Note that the integral in part a) is worked out in terms of by-now familiar special functions in eqn. B.36. The value of  $\Gamma(3/2)\zeta(3/2)$  is about 2.315. In part b), the volume of a mole of Fe is 7.11 cm<sup>3</sup> (see table on p. 404). Hint:  $T_0$  is about 4000K; find 2 more significant digits. In part c) you should find that the magnon and phonon contributions to  $C_V$  are equal at 2.17 K. You should further find that U is proportional to the same dimensional integral as in part a) but with 3/2 replacing 1/2 in the exponent of x in the numerator, so with value  $\Gamma(5/2)\zeta(5/2) \approx 1.783$ . In part d) just argue quickly that the number of magnons is proportional to the same integral with just 1 in the numerator (i.e. n = 0 in eqn. B.36. From the discussion of liquid <sup>4</sup>He we know that  $\zeta(1/2) = \infty$ , so you should be able to see this easily from the integral itself. This divergence implies that the material never magnetizes in the first place in 2D.
- 3. (10) 7.66 a,b,c Bose condensation of <sup>87</sup>Rb atoms. In part b), show that  $k_{\rm B}T_{\rm c} \propto N^{2/3}\epsilon_0$  and find the proportionality constant. If you crank through part d) [not assigned], you would find that the ratio of N<sub>0</sub>/N<sub>1</sub> is nearly 5 times as large as in part c).
- 4. (6) 7.70 a,b Heat capacity of gas of bosons. Compare your predicted value of  $C_V(T_c)$  with that in Fig. 7.37. Take note of the result in part c).
- 5. (10) 7.71. Hints: Start with your result for  $C_V(T)$  from the previous problem to find S(T). You should then find  $F \propto -N (T/T_c)^{3/2} k_B T$  and  $p \propto \lambda_T^{-3} k_B T$ . (You must verify these results and find the prefactors.)