Formulas for Final, PHYS404, Fall 2014 v1.2

$$\Omega(N,N_\uparrow)=\frac{N!}{N_\uparrow!\left(N-N_\uparrow\right)!}$$

$$\Omega(N,q)=\frac{(q+N-1)!}{q!\left(N-1\right)!}$$

$$\Omega(U,V,N)=f(N)V^N U^{fN/2}$$

$U=(f/2)\,Nk_BT$; determining f: 3 for atoms,
5 for diatomic molecules at low T, 7 at higher T,
2 for each direction of a[n Einstein] oscillator
Dimer rotational energy = $j\,(j+1)\,\varepsilon$

Ideal gas law $pV = Nk_BT = nRT$
van der Waals $pV = Nk_BT/(V-Nb)-aN^2/V^2$

Entropy $S = k_B \ln\Omega = \int dQ_{rev}/T$ $\quad 1/T = (\partial S/\partial U)_{V,N}$
Ideal gas $S = Nk_B\,[(V/(N\lambda_T^3) + 5/2]$; $\lambda_T=h/\sqrt{(2\pi m k_BT)}$
Paramag $S = Nk_B\,[\ln(2\cosh x) - x\tanh x]$; $x = \mu B/k_BT$
$U = -N\mu B\tanh(\beta\mu B)$; $\quad M = N\mu\tanh(\beta\mu B)$

$\Delta U = Q + W_{on} = Q - W_{by}$ $\qquad \Delta S_{tot} \geq 0$

Constants: $k_B \approx 10^{-4}$ eV/K = 1.38×10^{-23} J/K
C_V of 1 gm of water (ice) is 1 cal/K (~½ cal/K)
$R \sim 8.3$J/K $\qquad N_A = 6.02\times10^{23}$
Stirling: $\ln n! \approx n\ln n - n$

Expansions in $\varepsilon \ll 1$:
$\ln(1\pm\varepsilon) \approx \pm\varepsilon\,[-\varepsilon^2/2]$, $\exp(\pm\varepsilon) \approx 1\pm\varepsilon\,[+\varepsilon^2/2!]$
$\Gamma(n+1)=n!$ (integer n); $\Gamma(1/2)=\sqrt{\pi}$; $\Gamma(x+1)=x\Gamma(x)$

Change in internal energy, change in temperature, heat, work, during "simple" processes:
isobaric ($\Delta p = 0$), isochoric ($\Delta V = 0 = W$), isothermal ($\Delta U = \Delta T = 0$), adiabatic ($Q = 0$).
Along an isobar, $W_{by} = p(V_f - V_i)$; along an isotherm $W_{by} = Nk_BT\ln(V_f/V_i)$
Along an adiabat pV^γ is constant, as is (using the ideal gas law) $TV^{\gamma-1}$ Note $\gamma= (f+2)/f$

Helmholtz and Gibbs free energies $\quad F(T,V,[N]) = U - TS + \mu N$ $\qquad G(T,p,[N]) = U + pV - TS + \mu N$
$U(V,S\,[,N])$ $\quad H(p,S\,[,N])$ $\qquad \Phi = U - TS - \mu N$

Thermo. identities: $dU = TdS - p\,dV + \mu dN$
$dF = -S\,dT - p\,dV + \mu\,dN$
$dG = -S\,dT + V\,dp + \mu\,dN$, etc.

Maxwell relations: 2nd deriv of thermo functions do not depend on order of derivs

$\Delta S_{mix} = -Nk_B[x\ln x + (1-x)\ln(1-x)]$

Expansion $\beta = V^{-1}\,\partial V/\partial T|_p$ $\qquad \kappa_T = -V^{-1}\,\partial V/\partial p|_T$
$S = \int C/T\,dT$ $\qquad F = -k_BT\ln Z$ $\qquad U = -\partial\ln Z/\partial\beta$
$Z_N = Z_1^N/N!$ for indistinguishable $\quad \mu = (\partial F/\partial N)_{T,V}$

Nonrelativistic gas $Z_1 = (V/\lambda_T^3)\,Z_{int}$; $\lambda_T = h(2\pi m k_BT)^{-1/2}$

$Z = \Sigma_i\, g_i\, e^{-\beta\varepsilon i}$ $\qquad \mathcal{Z} = \Sigma_i\, g_i\, e^{-\beta(\varepsilon i - \mu Ni)} = \Sigma_N\, z^N Z_N$
[NB εi means ε_i]

$Z_{rot} \approx k_BT/2\varepsilon$ for $k_BT \gg \varepsilon$, dimer of identical atoms

Maxwell 3D $\quad D(v)=\left(\frac{m}{2\pi k_BT}\right)^{3/2} 4\pi v^2\exp\left(-\frac{mv^2}{2k_BT}\right)$

Ideal gas $\Phi = -k_BT\ln\mathcal{Z} = -pV$
$d\Phi = -S\,dT - p\,dV - N\,d\mu$

$\mathcal{Z}_{FD/BE} = (1 \pm \exp[-\beta(\varepsilon-\mu)])^{\pm1}$ $\qquad \beta = 1/k_BT$

$$\bar{n}_{\frac{FD}{BE}}(\varepsilon;T,\mu)=\frac{1}{\exp[\beta(\varepsilon-\mu)]\pm1}=\frac{1}{z^{-1}\exp(\beta\varepsilon)\pm1}$$

$\varepsilon \propto n^s$: $\quad \varepsilon=\frac{h^2}{8mL^2}\left(n_x^2+n_y^2+n_z^2\right)=\frac{h^2n^2}{8mL^2}$ or $\varepsilon=\frac{hcn}{2L}$

$E_F = (\hbar^2/2m)(3\pi^2N_{el}/V)^{2/3}$ $\qquad \omega_D = (6\pi^2N_{at}/V)^{1/3}$

$\mathcal{G}(\varepsilon) \propto \varepsilon^{(D/s)-1}/\,\varepsilon_0^{(D/s)}$

$$\int^\infty H(\varepsilon)\bar{n}_{FD}(\varepsilon;T)d\varepsilon=\int_0^{\varepsilon_F}H(\varepsilon)d\varepsilon+\frac{\pi^2}{6}\left(k_BT\right)^2\left[H'(\varepsilon_F)-H(\varepsilon_F)\frac{\mathcal{G}'(\varepsilon_F)}{\mathcal{G}(\varepsilon_F)}\right]$$

[e.g.: $H = \mathcal{G}\,h$] $\quad \rightarrow +(\pi^2/6)(k_BT)^2\mathcal{G}(\varepsilon_F)h'(\varepsilon_F)$

Photons $u(\varepsilon) = [8\pi\varepsilon^3/(hc)^3]\;\bar{n}_{BE}(\varepsilon;T,0)$ $\qquad \varepsilon = hf$

$U/V = (8\pi^5/15)[(k_BT)^4/(hc)^3]$

Wien: $\lambda_{max}T = 0.0029$ m K; $\nu_{max} \neq c/\lambda_{max}$

Power from perfect blackbody radiator: $\sigma\times$area$\times T^4$

Debye $\quad T_D = (\hbar\,c_s/k_B)(6\pi^2N/V)^{1/3}$

$$U=\frac{9Nk_BT^4}{T_D^3}\int_0^{T_D/T}\frac{x^3}{e^x-1}dx$$

BEC $\quad \Gamma(\nu)Li_\nu(z)=\int_0^\infty\frac{x^{\nu-1}dx}{z^{-1}e^x-1}$ $\qquad N = (V/\lambda_T^3)Li_{3/2}(z)$

$U=(3/2)(V/\lambda_T^3)k_BT\,Li_{5/2}(z)$ $\quad N/V = 2.612/(\lambda_T(T=T_c))^3$

$E_{Ising} = -J\,\Sigma_{<i,j>}\,s_i\,s_j$, $\;s_i = \pm1$ $\quad$ MeanField $\bar{s} = \tanh(\beta qJ\bar{s})$