QUANTUM MECHANICS II

PROBLEM SET 9

due November 26, before class

I. THERE'S NO HOMEWORK WITHOUT AN ANHARMONIC OSCILLATOR

Consider an anharmonic oscillator in which the anharmonic part $\lambda\hat{x}^4$ is turned on at time $t = 0$ and turned of at time t'. The hamiltonian for the system is

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 + \theta(t)\theta(t' - t)\lambda\hat{x}^4. \tag{1}$$

Initially, at $t = 0$ the system is in its ground state. At $t > t'$ the energy of the system is measured. What are the possible outcomes of such a measurement and what are their probabilities in the leading order in a λ expansion? If the system can go from the ground state to an excited state, is there a contradiction with the conservation of energy principle?

II. TWO BOSONS IN A BOX

Two identical, non-interacting spin-0 particles are in the ground state of a three-dimensional box of sides L. At time $t = 0$ a $\delta(\mathbf{r}_1 - \mathbf{r}_2) = \delta(x_1 - x_2)\delta(y_1 - y_2)\delta(z_1 - z_2)$ potential (that is, $\langle \mathbf{r}'_1, \mathbf{r}'_2 |\hat{V}| \mathbf{r}_1, \mathbf{r}_2\rangle = \lambda\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}'_1)\delta(\mathbf{r}_2 - \mathbf{r}'_2)$) between them is turned on. What is the probability, in leading order of perturbation theory, for the particles to go to one of the first excited states (the 1st excited state of the unperturbed hamiltonian is degenerate so you have to sum over all of them)?