INTRODUCTION TO QUANTUM MECHANICS II

PROBLEM SET 8, due on Nov. 19

I. BORN RULE AGAIN (NOT TO BE CONFUSED WITH BORN AGAIN RULE)

The z component of the spin of an electron is measured and the result $S_z = \hbar/2$ is obtained. Immediately after, the component of the spin along the direction $\mathbf{n} = \frac{\sqrt{3}}{2}\mathbf{e}_x + \frac{1}{2}\mathbf{e}_z$ is measured. The operator corresponding to this last observable is given by $\mathbf{n}.\hat{\mathbf{S}}$. What are the possible outcomes of this measurement and their probabilities?

II. QUARKONIUM

Particles like protons and neutrons are made up of spin-1/2 fermions called quarks (and their anti-particles, the anti-quarks. The force between quarks is very strong (in fact, it is called the "strong nuclear force"). There are six kinds of quarks (called "flavors"): up, down, strange, charm, bottom and top, in increasing order of mass. The force between a quark and an anti-quark is essentially constant (about 16 tons !) for distances larger than about 10^{-16} m so the potential describing it is

$$V(r) = \sigma r, \tag{1}$$

where r is the radial distance between them and σ (called the string tension) is about 900 Mev/fm (MeV=10^6 eV and fm=10^{-15} m).

i) Use a simple variational argument (like we did in class) to *estimate* the binding energy of quarkonium. Verify that the bound states become more non-relativistic as the mass of the quarks is increased.

ii) Plugging the numbers one can verify that for the charm-anticharm and bottom-antibottom bound states are non-relativistic (the top is even heavier but decays too fast to form any bound state). Thus, we can use non-relativistic quantum mechanics to study them.

iii) Use the Bohr-Sommerfeld quantization condition to determine the bound state energies of a quarkonim made of (anti)quarks of mass M. The analytic expression is sufficient, no need to plug the numerical values. Consider only the s-wave states.

iv) Do you expect the results from iii) to be correct for low lying or very excited states?

III. WKB FOR CRAZY POTENTIAL

Find the first 10 energy levels of the hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} - \frac{\lambda}{\cosh^2(\hat{x}/L)}, \tag{2}$$

using the WKB approximation. Hint: to compute the integral, take a derivative in relation to the energy E, compute the integral and then integrate in relation to E. Plot the potential and the ground state wave function.