QUANTUM PHYSICS II

PROBLEM SET 5

due October 13, before class

A. Born rule for momentum probabilities

A spinless particle moves in one dimension and, at some instant, is described by the wave function $\psi(x) = \langle x|\psi\rangle$. At that instant the momentum of the particle is measured. What are the possible outcomes of this measurement and with which probabilities (probability densities, to be more precise) ?

B. Spin singlet

Consider two spin 1/2 particles. Show that the state $|singlet\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ is an eigenstate of both $\hat{\mathbf{S}}^2$ and $\hat{S}_z$ ($\hat{\mathbf{S}}$ is the *total* spin operator). What are the corresponding eigenvalues?

C. Warming up to the Bell inequalities

One of the deepest results obtained by the Human Race is the "Bell inequality" which I hope to discuss at some point at the end of the semester. The proof hinges on a simple fact about spin-1/2 particles. Suppose two spin-1/2 particles are known to be in a spin singlet state $|singlet\rangle$. Let $\hat{S}_a^{(1)} = \hat{\mathbf{S}}^{(1)}.\mathbf{a}$ be the component of the spin of particle one in the direction of the unit vector $\mathbf{a}$ and similarly for $\hat{S}_b^{(2)}$. Then,

$$\langle singlet|\hat{S}_a^{(1)}\hat{S}_b^{(2)}|singlet\rangle = -\frac{\hbar^2}{4}\cos\theta, \tag{1}$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$. Prove the relation above.

D. More bra-ketology

Consider a spinless particle moving in one dimension. Write the expressions below in the eigenbasis of position (that is, in terms of $\psi(x) = \langle x\,|\psi\rangle$):

i) $V(\hat{x})|\psi\rangle$, where $V(x)$ is an analytic function of x.

ii) $\hat{p}|\psi\rangle$

iii) $\left[\frac{\hat{p}^2}{2m} + V(\hat{x})\right]|\psi\rangle = E|\psi\rangle$

Write the expressions below in the eigenbasis of momentum (that is, in terms of $\tilde{\psi}(p) = \langle p\,|\psi\rangle$):

i) $\hat{x}^2|\psi\rangle$, where $V(x)$ is an analytic function of x.

ii) $\hat{p}|\psi\rangle$

iii) $\left[\frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2\right]|\psi\rangle = E|\psi\rangle$