In[437]:=
```
(*Ansatz:*)
f = Exp[-α x^2]
(*normalization*)
A = 1/Sqrt[Integrate[f^2, {x, -∞, ∞}, Assumptions → {α > 0, β > 1}]]
ψ = A f
(*check normalization*)
Simplify[Integrate[ψ^2, {x, -∞, ∞}, Assumptions → α > 0]]
(*compute <ψ|H|ψ>, using λ=m=1*)
En = Integrate[1/2 (D[ψ, x])^2 + x^4 ψ^2, {x, -∞, ∞}, Assumptions → {α > 0, β > 1}]
(*minimize*)
min = FindMinimum[En, {α}]
Print["E0 = ", min[[1]], "(λ hbar^4/m^2)^(1/3)"]
(*plot the wavefunction*)
Plot[(ψ /. min[[2]]), {x, -3, 3}]
```

Out[437]= $e^{-x^2 \alpha}$

Out[438]= $\left(\frac{2}{\pi}\right)^{1/4} \alpha^{1/4}$

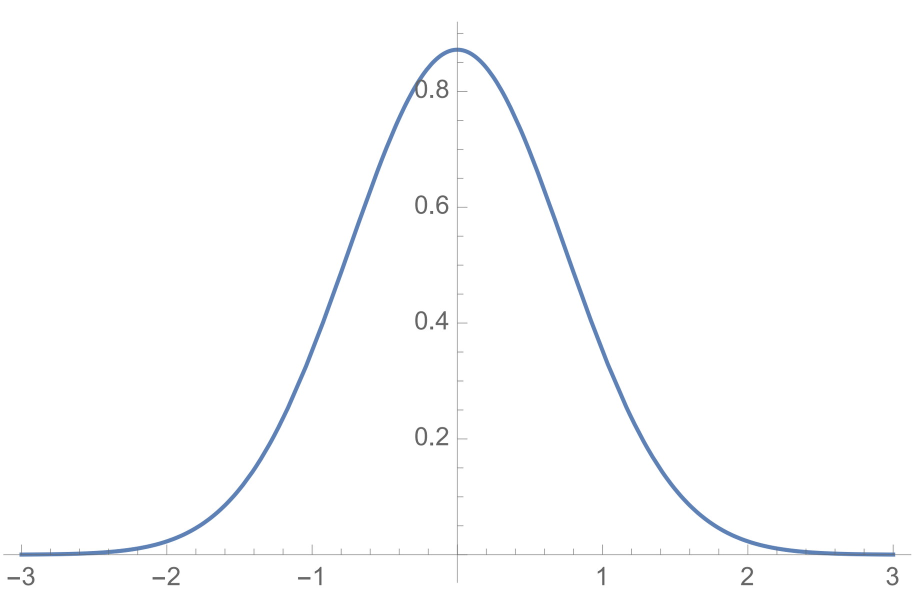
Out[439]= $e^{-x^2 \alpha} \left(\frac{2}{\pi}\right)^{1/4} \alpha^{1/4}$

Out[440]= 1

Out[441]= $\frac{3}{16 \alpha^2} + \frac{\alpha}{2}$

Out[442]= {0.68142, {α → 0.90856}}

E_0 = 0.68142(λ hbar^4/m^2)$^{1/3}$

Out[444]=


In[445]:=
```
(*Ansatz:*)
f = Exp[-α x^2] (a + b x^2 + c x^4 + d x^6 + e x^8)
(*normalization*)
A = 1/Sqrt[Integrate[f^2, {x, -∞, ∞}, Assumptions → {α > 0, β > 1}]]
ψ = A f
(*check normalization*)
Simplify[Integrate[ψ^2, {x, -∞, ∞}, Assumptions → α > 0]]
(*compute <ψ|H|ψ>, using λ=m=1*)
En = Integrate[1/2 (D[ψ, x])^2 + x^4 ψ^2, {x, -∞, ∞}, Assumptions → {α > 0, β > 1}]
(*minimize*)
min = FindMinimum[En, {α, a, b, c, d, e}]
Print["E0 = ", min[[1]], "(λ hbar^4/m^2)^(1/3)"]
(*plot the wavefunction*)
Plot[(ψ /. min[[2]]), {x, -3, 3}]
```

Out[445]= $e^{-x^2 \alpha} (a + b x^2 + c x^4 + d x^6 + e x^8)$

Out[446]= $$\left(256 \left(\frac{2}{\pi}\right)^{1/4}\right) \Big/ \left(\sqrt{\left(\frac{1}{\alpha^{17/2}} \left(2027025 e^2 + 16 \alpha^2 \left(10395 d^2 + 7560 c d \alpha + 1680 (c^2 + 2 b d) \alpha^2 + 1920 (b c + a d) \alpha^3 + 768 (b^2 + 2 a c) \alpha^4 + 2048 a b \alpha^5 + 4096 a^2 \alpha^6\right) + 840 e \alpha (1287 d + 4 \alpha (99 c + 4 \alpha (9 b + 4 a \alpha)))\right)\right)}\right)$$

Out[447]= $$\left(256 e^{-x^2 \alpha} \left(\frac{2}{\pi}\right)^{1/4} (a + b x^2 + c x^4 + d x^6 + e x^8)\right) \Big/ \left(\sqrt{\left(\frac{1}{\alpha^{17/2}} \left(2027025 e^2 + 16 \alpha^2 \left(10395 d^2 + 7560 c d \alpha + 1680 (c^2 + 2 b d) \alpha^2 + 1920 (b c + a d) \alpha^3 + 768 (b^2 + 2 a c) \alpha^4 + 2048 a b \alpha^5 + 4096 a^2 \alpha^6\right) + 840 e \alpha (1287 d + 4 \alpha (99 c + 4 \alpha (9 b + 4 a \alpha)))\right)\right)}\right)$$

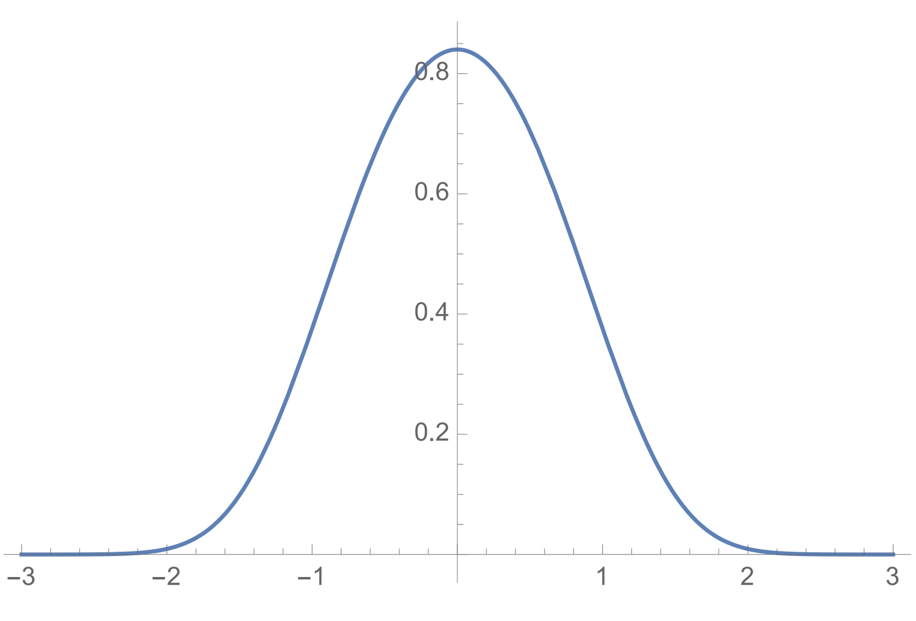
Out[448]= 1

Out[449]= $$\left(654729075 e^2 + 275675400 d e \alpha + 32432400 (d^2 + 2 c e) \alpha^2 + 1081080 (16 c d + e (16 b + 31 e)) \alpha^3 + 665280 (4 c^2 + 8 b d + 8 a e + 23 d e) \alpha^4 + 120960 (16 b c + 16 a d + 23 d^2 + 14 c e) \alpha^5 + 107520 (4 b^2 + 8 a c + 15 c d - 17 b e) \alpha^6 + 30720 (16 a b + 15 c^2 - 2 b d - 98 a e) \alpha^7 + 49152 (4 a^2 + 7 b c - 25 a d) \alpha^8 + 32768 (7 b^2 - 18 a c) \alpha^9 - 262144 a b \alpha^{10} + 524288 a^2 \alpha^{11}\right) \Big/ \left(16 \alpha^2 \left(2027025 e^2 + 16 \alpha^2 \left(10395 d^2 + 7560 c d \alpha + 1680 (c^2 + 2 b d) \alpha^2 + 1920 (b c + a d) \alpha^3 + 768 (b^2 + 2 a c) \alpha^4 + 2048 a b \alpha^5 + 4096 a^2 \alpha^6\right) + 840 e \alpha (1287 d + 4 \alpha (99 c + 4 \alpha (9 b + 4 a \alpha)))\right)\right)$$

FindMinimum ::lstol :
The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function . You may need more than MachinePrecision digits of working precision to meet these tolerances . ≫

Out[450]= {0.667987, {α → 2.23911, a → 1.12835, b → 1.76954, c → 1.2481, d → 0.42701, e → 0.160639}}

E_0 = 0.667987(λ hbar^4/m^2)$^{1/3}$

Out[452]=


In[501]:=
```
(*Ansatz:*)
f = Exp[-α x^(2 β)]
(*normalization*)
A = 1/(Sqrt[2 Integrate[f^2, {x, 0, ∞}, Assumptions → {α > 0, β > 1}]])
ψ = A f
(*check normalization*)
Simplify[2 Integrate[ψ^2, {x, 0, ∞}, Assumptions → {α > 0, β > 1}]]
(*compute <ψ|H|ψ>, using λ=m=1*)
En = 2 Integrate[1/2 (D[ψ, x])^2 + x^4 ψ^2, {x, 0, ∞}, Assumptions → {α > 0, β > 1}]
(*minimize*)
min = FindMinimum[En, {α, β}]
Print["E0 = ", min[[1]], "(λ hbar^4/m^2)^(1/3)"]
(*plot the wavefunction*)
Plot[(ψ /. min[[2]]), {x, 0, 3}]
```

Out[501]= $e^{-x^{2\beta} \alpha}$

Out[502]= $$\frac{1}{\sqrt{2^{1-\frac{1}{2\beta}} \alpha^{-\frac{1}{2}/\beta} \text{Gamma}\left[1 + \frac{1}{2\beta}\right]}}$$

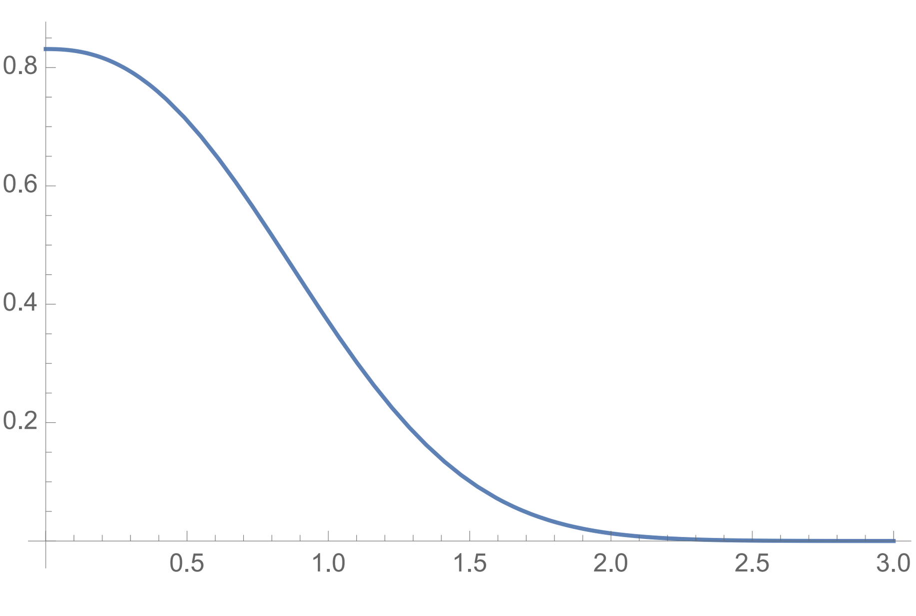
Out[503]= $$\frac{e^{-x^{2\beta} \alpha}}{\sqrt{2^{1-\frac{1}{2\beta}} \alpha^{-\frac{1}{2}/\beta} \text{Gamma}\left[1 + \frac{1}{2\beta}\right]}}$$

Out[504]= 1

Out[505]= $$2 \left(\frac{2^{-3+\frac{1}{\beta}} \alpha^{\frac{1}{\beta}} \beta \,\text{Gamma}\left[2 - \frac{1}{2\beta}\right]}{\text{Gamma}\left[1 + \frac{1}{2\beta}\right]} + \frac{2^{-\frac{2+\beta}{\beta}} \alpha^{-2/\beta} \text{Gamma}\left[\frac{5}{2\beta}\right]}{\text{Gamma}\left[\frac{1}{2\beta}\right]}\right)$$

Out[506]= {0.66933, {α → 0.807838, β → 1.18346}}

E_0 = 0.66933(λ hbar^4/m^2)$^{1/3}$

Out[508]=


In[493]:=
```
(*Ansatz:*)
f = Exp[-α x^(2 β)] (1 + a x^2 + b x^4)
(*normalization*)
A = 1/(Sqrt[2 Integrate[f^2, {x, 0, ∞}, Assumptions → {α > 0, β > 1}]])
ψ = A f
(*check normalization*)
Simplify[2 Integrate[ψ^2, {x, 0, ∞}, Assumptions → {α > 0, β > 1}]]
(*compute <ψ|H|ψ>, using λ=m=1*)
En = 2 Integrate[1/2 (D[ψ, x])^2 + x^4 ψ^2, {x, 0, ∞}, Assumptions → {α > 0, β > 1}]
(*minimize*)
min = FindMinimum[En, {α, β, a, b, c}]
Print["E0 = ", min[[1]], "(λ hbar^4/m^2)^(1/3)"]
(*plot the wavefunction*)
Plot[(ψ /. min[[2]]), {x, 0, 3}]
```

Out[493]= $e^{-x^{2\beta} \alpha} (1 + a x^2 + b x^4)$

Out[494]= $$\left(3\sqrt{5}\right) \Big/ \left(\sqrt{\left(\frac{1}{\beta} 2^{1-\frac{9}{2\beta}} \alpha^{-\frac{9}{2}/\beta} \left(45 \times 16^{\frac{1}{\beta}} \alpha^{4/\beta} \beta \,\text{Gamma}\left[1 + \frac{1}{2\beta}\right] + 9 \times 4^{\frac{1}{\beta}} (a^2 + 2 b) \alpha^{2/\beta} \beta \,\text{Gamma}\left[1 + \frac{5}{2\beta}\right] + 5 \left(b^2 \beta \,\text{Gamma}\left[1 + \frac{9}{2\beta}\right] + 9 \times 8^{\frac{1}{\beta}} a \alpha^{3/\beta} \text{Gamma}\left[\frac{3}{2\beta}\right] + 9 \times 2^{\frac{1}{\beta}} a b \alpha^{\frac{1}{\beta}} \text{Gamma}\left[\frac{7}{2\beta}\right]\right)\right)\right)}\right)$$

Out[495]= $$\left(3\sqrt{5}\, e^{-x^{2\beta} \alpha} (1 + a x^2 + b x^4)\right) \Big/ \left(\sqrt{\left(\frac{1}{\beta} 2^{1-\frac{9}{2\beta}} \alpha^{-\frac{9}{2}/\beta} \left(45 \times 16^{\frac{1}{\beta}} \alpha^{4/\beta} \beta \,\text{Gamma}\left[1 + \frac{1}{2\beta}\right] + 9 \times 4^{\frac{1}{\beta}} (a^2 + 2 b) \alpha^{2/\beta} \beta \,\text{Gamma}\left[1 + \frac{5}{2\beta}\right] + 5 \left(b^2 \beta \,\text{Gamma}\left[1 + \frac{9}{2\beta}\right] + 9 \times 8^{\frac{1}{\beta}} a \alpha^{3/\beta} \text{Gamma}\left[\frac{3}{2\beta}\right] + 9 \times 2^{\frac{1}{\beta}} a b \alpha^{\frac{1}{\beta}} \text{Gamma}\left[\frac{7}{2\beta}\right]\right)\right)\right)}\right)$$

Out[496]= 1

Out[497]= $$\left(5 \times 2^{-4-\frac{2}{\beta}} \alpha^{-2/\beta} \beta \left(117 \times 2^{2+\frac{7}{\beta}} \alpha^{7/\beta} \beta \,\text{Gamma}\left[2 - \frac{1}{2\beta}\right] + 117 \times 2^{2+\frac{6}{\beta}} a \alpha^{6/\beta} (-3 + 2\beta) \text{Gamma}\left[1 + \frac{1}{2\beta}\right] + 13 \times 2^{4+\frac{2}{\beta}} (a^2 + 2 b) \alpha^{2/\beta} \text{Gamma}\left[1 + \frac{9}{2\beta}\right] + 144 b^2 \,\text{Gamma}\left[1 + \frac{13}{2\beta}\right] + \frac{117 \times 32^{\frac{1}{\beta}} \alpha^{5/\beta} \left(6 b (-5 + 2\beta) + a^2 (1 + 6\beta)\right) \text{Gamma}\left[\frac{3}{2\beta}\right]}{\beta} + \frac{117 \times 2^{\frac{4+\beta}{\beta}} \alpha^{4/\beta} (4 + a b (-3 + 10\beta)) \text{Gamma}\left[\frac{5}{2\beta}\right]}{\beta} + \frac{117 \times 8^{\frac{1}{\beta}} \alpha^{3/\beta} \left(16 a + b^2 (1 + 14\beta)\right) \text{Gamma}\left[\frac{7}{2\beta}\right]}{\beta} + \frac{117 \times 2^{4+\frac{1}{\beta}} a b \alpha^{\frac{1}{\beta}} \text{Gamma}\left[\frac{11}{2\beta}\right]}{\beta}\right)\right) \Big/ \left(13 \left(45 \times 16^{\frac{1}{\beta}} \alpha^{4/\beta} \beta \,\text{Gamma}\left[1 + \frac{1}{2\beta}\right] + 9 \times 4^{\frac{1}{\beta}} (a^2 + 2 b) \alpha^{2/\beta} \beta \,\text{Gamma}\left[1 + \frac{5}{2\beta}\right] + 5 \left(b^2 \beta \,\text{Gamma}\left[1 + \frac{9}{2\beta}\right] + 9 \times 8^{\frac{1}{\beta}} a \alpha^{3/\beta} \text{Gamma}\left[\frac{3}{2\beta}\right] + 9 \times 2^{\frac{1}{\beta}} a b \alpha^{\frac{1}{\beta}} \text{Gamma}\left[\frac{7}{2\beta}\right]\right)\right)\right)$$

FindMinimum ::lstol :
The line search decreased the step size to within the tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the function . You may need more than MachinePrecision digits of working precision to meet these tolerances . ≫

Out[498]= {0.668018, {α → 1.69461, β → 1.01987, a → 0.940747, b → 0.49727, c → 1.}}

E_0 = 0.668018(λ hbar^4/m^2)$^{1/3}$

Out[500]=