Phys 402

Spring 2019

Homework 0

Due Friday, February 1, 2019 @ 9 AM

These are (some of the) physics and math skills that you will need for Phys 402. Review any concepts that present difficulty. Complete the following by hand (no assistance from computers!):

1. Use the Euler formula to expand $e^{i\theta}$ for real θ.
2. Given the three Cartesian unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$, calculate the following:
   a. $\hat{x} \times \hat{y}$
   b. $|\hat{x}|$
   c. $\hat{x} \cdot \hat{y}$
3. Given the vectors $\vec{r} = (r_x, r_y, r_z)$ and $\vec{s} = (s_x, s_y, s_z)$, calculate the cross product vector $\vec{r} \times \vec{s}$ in terms of its Cartesian components.
4. Find the eigenvalues and eigenvectors of this matrix: $\bar{\bar{A}} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. [For a review of linear algebra, see Appendix A of Griffiths.]
5. What is the determinant of $\bar{\bar{A}}$ and how is it related to the eigenvalues?
6. What is the trace of $\bar{\bar{A}}$ and how is it related to the eigenvalues?
7. Given a well-behaved scalar function of position $\chi(\vec{r})$ (e.g. the temperature distribution on the surface of the earth), what can we say is always true about the curl of the gradient of χ?
8. Given a vector field $\vec{F} = k(x, 2y^2, 3z^3)$, where k is a constant, calculate its curl, $\nabla \times \vec{F}$. If F is a physical field (flow, force, etc.), what is the physical interpretation of $\nabla \times \vec{F}$?
9. Calculate the vector divergence of $\vec{F}$, namely $\vec{\nabla} \cdot \vec{F}$. What is the physical interpretation of this vector divergence?
10. What is the general solution to the second-order linear differential equation $\ddot{x} = -\omega^2 x$, where ω is a real positive number?
11. What is the general solution to the second-order linear differential equation $\ddot{x} = +k^2 x$, where k is a real positive number?
12. Given $ln(y) = b\ ln(x)$, where b is a constant, find y as a function of x, $y(x)$.
13. Evaluate the integral $I = \int_{-2}^{3} 5x\ dx$.
14. Expand $y(x) = \ln(1 + x)$ to second order for $x \ll 1$. Write the series expansion for $y(x) = \frac{1}{1-x}$ valid for $-1 < x < 1$.

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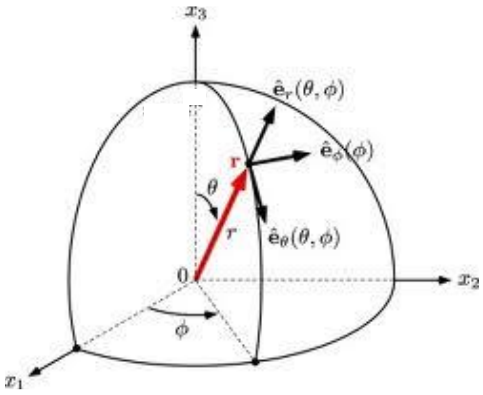
15. Write down the differential volume element d^3r in spherical coordinates (r, θ, ϕ). Use the figure below for definition of the spherical coordinates.



16. What is wrong with this equation ($\mathcal{H}$ is a Hamiltonian, e is the electronic charge, E_0 is electric field magnitude)? $\mathcal{H} = eE_0z\hat{z}$