Physics 402
Spring 2019
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Formula Sheet

Infinite square well $\mathcal{H}^0 = \frac{p^2}{2m} + V(x)$ $\quad V(x) = \begin{cases} 0 \text{ for } 0 < x < a \\ \infty \text{ for x<0 and x>a} \end{cases}$ $\quad E_n^0 = \frac{n^2\pi^2\hbar^2}{2ma^2}$

and $\psi^0(x) = \begin{cases} \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \text{ for } 0 < x < a \\ 0 \text{ for x<0 and x>a} \end{cases}$ with $n = 1, 2, 3, \ldots$

Harmonic Oscillator $H^0 = \frac{p_x^2}{2m} + \frac{k}{2}x^2$, $\psi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}} H_n(\xi) e^{-\xi^2/2}$, $\xi \equiv \sqrt{\frac{m\omega}{\hbar}}x$,

$E_n = (n + \frac{1}{2})\hbar\omega$; $n = 0, 1, 2, \ldots$; $x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$; $p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$;

$a_+\psi_n = \sqrt{n+1}\psi_{n+1}$; $a_-\psi_n = \sqrt{n}\psi_{n-1}$

Hydrogen Atom $\mathrm{H}^0 = -\frac{\hbar^2}{2m}\nabla^2 + \frac{(-e)(+e)}{4\pi\varepsilon_0 r}$, $n = 1, 2, 3, \ldots$, $\ell = 0, 1, \ldots, n-1$, $-\ell \le m \le \ell$, $\psi_{n,\ell,m}^0(r,\theta,\varphi) = Const_{n,\ell}\, e^{-\frac{r}{na}} \left(\frac{2r}{na}\right)^{\ell} L_{n-\ell-1}^{2\ell+1}\left(\frac{2r}{na}\right) Y_\ell^m(\theta,\phi)$; $L^2|\ell\ m_\ell\rangle = \ell(\ell+1)\hbar^2|\ell\ m_\ell\rangle$,

$L_z|\ell\ m_\ell\rangle = m_\ell\hbar|\ell\ m_\ell\rangle$; $S^2|s\ m_s\rangle = s(s+1)\hbar^2|s\ m_s\rangle$, $S_z|s\ m_s\rangle = m_s\hbar|s\ m_s\rangle$;
$J^2|j\ m_j\rangle = j(j+1)\hbar^2|j\ m_j\rangle$, $J_z|j\ m_j\rangle = m_j\hbar|j\ m_j\rangle$

Perturbation theory $\mathrm{H}^0\psi_n^0 = E_n^0\psi_n^0$, $H = H^0 + H'$, $\mathrm{H}\psi_n = E_n\psi_n$

$\psi_n = \psi_n^0 + \lambda\psi_n^1 + \lambda^2\psi_n^2 + \ldots$; $E_n = E_n^0 + \lambda E_n^1 + \lambda^2 E_n^2 + \ldots$;

$$E_n^1 = <\psi_n^0|\mathrm{H}'|\psi_n^0> = \int \psi_n^{0*}\,\mathrm{H}'\,\psi_n^0\, d^3r,\quad \psi_n^1 = \sum_{\ell\neq n}\left(\frac{\int \psi_\ell^{0*}\,\mathrm{H}'\,\psi_n^0\, d^3r}{E_n^0 - E_\ell^0}\right)\psi_\ell^0,$$

$$E_n^2 = \sum_{k\neq n}\frac{\left|\int \psi_k^{0*}\,\mathrm{H}'\,\psi_n^0\, d^3r\right|^2}{E_n^0 - E_k^0},\quad \alpha \equiv \frac{e^2}{4\pi\varepsilon_0\hbar c} \cong \frac{1}{137.036}$$

$\overline{\overline{W}}\vec{\alpha} = E^1\vec{\alpha}$ $\quad W_{k,j} \equiv \langle\psi_k^0|H'|\psi_j^0\rangle$ $\quad \mathrm{H}_{so} = -\vec{\mu}\bullet\vec{B}$; $\vec{S}\bullet\vec{L} = \frac{1}{2}\left(J^2 - L^2 - S^2\right)$;

Spin-orbit, Zeeman, addition of angular momenta

$$\left|j\ m_j\right\rangle = \sum_{m_\ell + m_s = m_j} C_{m_\ell\ m_s\ m_j}^{\ell\ \ s\ \ j}\left|\ell\ m_\ell\right\rangle\left|s\ m_s\right\rangle;\ \vec{\mu}_{Total} = \vec{\mu}_\ell + \vec{\mu}_S = -\frac{e}{2m}\left(\vec{L} + 2\vec{S}\right)$$

$\mathcal{H}_Z^1 = -\vec{\mu}_{Total}\cdot\vec{B}_{ext}$ $\quad E_Z^1 = \frac{e}{2m}\vec{B}_{ext}\cdot\langle\vec{J} + \vec{S}\rangle$ $\quad E_Z^1 = \mu_B g_J B_{ext} m_J$ $\quad \mu_B = \frac{e\hbar}{2m}$;

$g_J = 1 + \frac{j(j+1) - \ell(\ell+1) + s(s+1)}{2j(j+1)}$ $\quad E_{Z,strong}^1 = \frac{e}{2m}\vec{B}_{ext}\cdot\langle\vec{L} + 2\vec{S}\rangle = \mu_B B_{ext}(m_\ell + 2m_s)$

$$\int \sin\theta\cos\theta\, d\theta = -\int \cos\theta\, d(\cos\theta);\quad \int_0^\pi \sin^2(nu)\sin(u)\,du = \frac{4n^2}{4n^2-1};\quad \int_0^\infty r^n e^{-r/a}\,dr = a^{n+1}\, n!$$