Physics 402
Spring 2019
Prof. Belloni
Discussion Worksheet for February 20, 2019

PLEASE CLOSED BOOK (MIDTERM SIMULATION?)

Consider a particle in a three-dimensional infinite cubical well:

$$V(x,y,z) = \begin{cases} 0;\ if\ 0 < x < a, 0 < y < a, and\ 0 < z < a \\ \infty;\ otherwise \end{cases}$$

The stationary states are:

$$\psi^0_{n_x n_y n_z} = \left(\frac{2}{a}\right)^{\frac{3}{2}} \sin\left(\frac{n_x \pi}{a} x\right) \sin\left(\frac{n_y \pi}{a} y\right) \sin\left(\frac{n_z \pi}{a} z\right)$$

where n_x, n_y and n_z are positive integers.
The corresponding allowed energies are:

$$E^0_{n_x n_y n_z} = \frac{\pi^2 \hbar^2}{2ma^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

What is the degeneracy of the ground state? What is the degeneracy of the first excited state?

Let us now consider the following perturbation:

$$H'(x,y,z) = \begin{cases} V_0;\ if\ 0 < x < \frac{a}{2}\ and\ 0 < y < \frac{a}{2} \\ 0;\ otherwise \end{cases}$$

What is the first-order correction to the energy of the ground state?
Under which constraint is this a small perturbation?
What is the first-order correction to the energy of the first excited state?
If there is a degeneracy, write the W matrix, diagonalize it and find the "good" unperturbed states. Is the perturbation lifting the degeneracy?

Useful integral (which I am not sure I will provide in the exam formula sheet: I will put sine/cosine addition formulae, though):

$$\int_0^{\frac{a}{2}} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{2\pi x}{a}\right) dx = \frac{2a}{3\pi}$$