Physics 402
Spring 2019
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Discussion Worksheet for February 6, 2019

1. The Clebsch-Gordan coefficients allow us to go back and forth between the "coupled" and "un-coupled" wavefunctions for multiple-spin systems. Consider two spin-1/2 particles described by kets $\left|\frac{1}{2}\ m_1\right\rangle$ and $\left|\frac{1}{2}\ m_2\right\rangle$. Using Table 4.8 of Griffiths, write down the triplet and singlet states of the coupled representation in terms of the uncoupled single-particle kets.

We have done this exercise already, use it as a warm-up for the following case: consider two spin-1 particles, and write down the coupled representation (what are the possible values of the total spin of this 2-particle system?) in terms of the uncoupled single-particle kets.

2. Consider again two spin-1/2 particles. Using Table 4.8 of Griffiths, write down the uncoupled single-particle wavefunctions in terms of the coupled kets $|s\ m_s\rangle$.

Once this warm-up is done, let's do the case with two spin-1 particles, but only a few of them: $|1\ 0\rangle|1\ 0\rangle$, $|1\ -1\rangle|1\ 1\rangle$ and $|1\ -1\rangle|1\ 1\rangle$ (the exercise consists in writing each of these three 2-particle kets as a combination of $|s\ m_s\rangle$, with the opportune values of s and m_s).

PDG

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

m_1 m_2	J M	J M	...
m_1 m_2	Coefficients		
. .			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

1/2×1/2

m_1 m_2	J=1, M=+1
+1/2 +1/2	1

m_1 m_2	J=1, M=0	J=0, M=0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	J=1, M=−1
−1/2 −1/2	1

1×1/2

m_1 m_2	J=3/2, M=+3/2
+1 +1/2	1

m_1 m_2	J=3/2, M=+1/2	J=1/2, M=+1/2
+1 −1/2	1/3	2/3
0 +1/2	2/3	−1/3

m_1 m_2	J=3/2, M=−1/2	J=1/2, M=−1/2
0 −1/2	2/3	1/3
−1 +1/2	1/3	−2/3

m_1 m_2	J=3/2, M=−3/2
−1 −1/2	1

2×1

m_1 m_2	J=3, M=+3
+2 +1	1

m_1 m_2	J=3, M=+2	J=2, M=+2
+2 0	1/3	2/3
+1 +1	2/3	−1/3

m_1 m_2	J=3, M=+1	J=2, M=+1	J=1, M=+1
+2 −1	1/15	1/3	3/5
+1 0	8/15	1/6	−3/10
0 +1	2/5	−1/2	1/10

m_1 m_2	J=3, M=0	J=2, M=0	J=1, M=0
+1 −1	1/5	1/2	3/10
0 0	3/5	0	−2/5
−1 +1	1/5	−1/2	3/10

m_1 m_2	J=3, M=−1	J=2, M=−1	J=1, M=−1
0 −1	2/5	1/2	1/10
−1 0	8/15	−1/6	−3/10
−2 +1	1/15	−1/3	3/5

m_1 m_2	J=3, M=−2	J=2, M=−2
−1 −1	2/3	1/3
−2 0	1/3	−2/3

m_1 m_2	J=3, M=−3
−2 −1	1

1×1

m_1 m_2	J=2, M=+2
+1 +1	1

m_1 m_2	J=2, M=+1	J=1, M=+1
+1 0	1/2	1/2
0 +1	1/2	−1/2

m_1 m_2	J=2, M=0	J=1, M=0	J=0, M=0
+1 −1	1/6	1/2	1/3
0 0	2/3	0	−1/3
−1 +1	1/6	−1/2	1/3

m_1 m_2	J=2, M=−1	J=1, M=−1
0 −1	1/2	1/2
−1 0	1/2	−1/2

m_1 m_2	J=2, M=−2
−1 −1	1

2×1/2

m_1 m_2	J=5/2, M=+5/2
+2 +1/2	1

m_1 m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+2 −1/2	1/5	4/5
+1 +1/2	4/5	−1/5

m_1 m_2	J=5/2, M=+1/2	J=3/2, M=+1/2
+1 −1/2	2/5	3/5
0 +1/2	3/5	−2/5

m_1 m_2	J=5/2, M=−1/2	J=3/2, M=−1/2
0 −1/2	3/5	2/5
−1 +1/2	2/5	−3/5

m_1 m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1 −1/2	4/5	1/5
−2 +1/2	1/5	−4/5

m_1 m_2	J=5/2, M=−5/2
−2 −1/2	1

3/2×1/2

m_1 m_2	J=2, M=+2
+3/2 +1/2	1

m_1 m_2	J=2, M=+1	J=1, M=+1
+3/2 −1/2	1/4	3/4
+1/2 +1/2	3/4	−1/4

m_1 m_2	J=2, M=0	J=1, M=0
+1/2 −1/2	1/2	1/2
−1/2 +1/2	1/2	−1/2

m_1 m_2	J=2, M=−1	J=1, M=−1
−1/2 −1/2	3/4	1/4
−3/2 +1/2	1/4	−3/4

m_1 m_2	J=2, M=−2
−3/2 −1/2	1

3/2×1

m_1 m_2	J=5/2, M=+5/2
+3/2 +1	1

m_1 m_2	J=5/2, M=+3/2	J=3/2, M=+3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	−2/5

m_1 m_2	J=5/2, M=+1/2	J=3/2, M=+1/2	J=1/2, M=+1/2
+3/2 −1	1/10	2/5	1/2
+1/2 0	3/5	1/15	−1/3
−1/2 +1	3/10	−8/15	1/6

m_1 m_2	J=5/2, M=−1/2	J=3/2, M=−1/2	J=1/2, M=−1/2
+1/2 −1	3/10	8/15	1/6
−1/2 0	3/5	−1/15	−1/3
−3/2 +1	1/10	−2/5	1/2

m_1 m_2	J=5/2, M=−3/2	J=3/2, M=−3/2
−1/2 −1	3/5	2/5
−3/2 0	2/5	−3/5

m_1 m_2	J=5/2, M=−5/2
−3/2 −1	1