1. Variational principle and the 1D harmonic oscillator. Make a guess for the ground state wavefunction of the 1D harmonic oscillator as a simple parabolic function:

$$\psi(x) = \begin{cases} A(a^2 - x^2) & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

As a first step, sketch the potential and wavefunction, and then find the normalization constant A.

2. Using the above wavefunction and the length scale 'a' as the variational parameter, find an upper bound for the ground state energy of the 1D harmonic oscillator.

Partial answer: the value of a that minimizes $\langle H \rangle$ is $a^2 = \sqrt{\dfrac{35}{2} \dfrac{\hbar}{m\omega}}$