Physics 402
Spring 2019
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Discussion Worksheet for May 1, 2019

Griffiths problem 8.17 (2nd edition) reports an interesting question, originally posted about ten years ago on Scientific American. The question was about the fact that some numbers are so large that they defy the ever increasing capabilities of modern computers. Let us consider a case in which we do expect a large number to enter the scene: let us calculate the probability that a beer can will tip over due to quantum tunneling.

Let us consider the can as a uniform cylinder; as it tips over, let *x* be the height of its center above the equilibrium position (1/2 of the can height). The potential energy, as *x* increase from 0 to a critical value x_0, is mgx. When *x* reaches the value $x_0 = \sqrt{R^2 + (h/2)^2} - (h/2)$, the can tips over. Let us:

1. Start considering the can as having energy 0, at $x = 0$. Sketch the potential, until the point x_0
2. At x_0, the stable position becomes the one where the can is laying on its side. This is equivalent to thinking that the potential sharply drops to 0 (we can also think of what happens here as x going to $-h/2$ through a different path). Calculate the tunneling probability in the WKB approximation
3. We can estimate the lifetime of the keep-standing state of the beer can using the expression that Gamow considered to define the lifetime of alpha decays. Consider the beer as a particle with mass m and average kinetic energy $k_B T$, that keeps running between $x = 0$ and $x = x_0$ points, and every time it reaches x_0 it has a probability $e^{-2\gamma}$ to tip over. Estimate the time it takes, on average, for a beer can to tip over due to quantum tunneling, at room temperature (hint: it is going to be a really big number)

Useful constants:

- Beer characteristics: 16cm height; 6cm diameter; 0.1kg weight
- $k_B T = 0.025 eV$ at room temperature
- $g = 9.8 m \cdot s^{-2}$
- $1 yr = \pi \cdot 10^7 s$
- $\hbar = 1.054 \cdot 10^{-34} m^2 \cdot kg \cdot s^{-1} = 6.6 \cdot 10^{-16} eV \cdot s$
- $1 eV = 1.8 \cdot 10^{-36} kg \cdot c^2$

Useful formula:

- $\gamma = \frac{1}{\hbar} \int^{x} |\sqrt{2m(E - V(x))}| dx'$