Physics 402
Spring 2019
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Discussion Worksheet for April 24, 2019

When we searched for the most probable configuration of a system with many particles, under the constraints that the total energy and the number of particles are fixed, we obtained an expression for the density of states that depended on two Lagrange multipliers, one per constraint. Let us calculate explicitly their value by imposing the two constraints on the expressions for the density of states we found. Let us start with the case of distinguishable particles. We found that:

$$n_s = g_s e^{-(\alpha+\beta E_s)}$$

Let us then:

1. Calculate the degeneracy g_s for a 3-dimensional ideal gas. Remember: it is easy if you move to the **k**-space, calculate the volume occupied by states with the same **k**, and divide that volume by the volume occupied by a single state
2. Assuming that the states are dense enough that sums can be replaced by integrals, and using the density of states just calculated, write the constraints on number of particles and energy in integral form
3. Compare the two expressions just calculated to the classical formula for the energy of an atom at temperature T: $\frac{E}{N} = \frac{3}{2} k_B T$ to obtain an expression for β
4. If you feel very adventurous, you can do the same for identical bosons/fermion systems. In that case, we have:

$$n_s = \frac{g_s}{e^{(\alpha+\beta E_s)} \pm 1}$$

The following integral could be useful:

$$\int_0^\infty \frac{x^{s-1}}{e^x - 1} dx = \Gamma(s)\zeta(s)$$

where the gamma function and the Riemann zeta function have tabulated values. In the case of fermions, the following integral can be used:

$$\int_0^\infty \frac{x^{s-1}}{e^{x-y} + 1} dx = \Gamma(s) F_s(y)$$

where $F_s(x)$ is the complete Fermi-Dirac integral. I suggest considering the special cases of a Fermi gas at temperature T=0, and of a gas of photons.

Another useful integral:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^4 e^{-\frac{x^2}{2}} dx = 3$$

An additional quick problem. Let us consider Helium gas. We can think of it as a set of particles that obey Maxwell-Boltzmann statistics (both Bose-Einstein and Fermi-Dirac tend to a Maxwell-Boltzmann when T is high – compared to?). It is easy to show that the density of states is proportional to k, and the energy is proportional to k^2.

1. Let us derive $g(E)dE$ from $g(k)dk$. Makes sure to use the correct energy expression (consider the gas as particles in an infinitely deep quantum well)
2. Using the fact that $hc = 1240eVnm$ and $mc^2 = 4000MeV$ for Helium, how many states exists with energy between 0.0086eV and 0.0088eV, at a temperature of 200K?
3. How does the number of states compare with the number of Helium atoms in a mole of gas?