

**Physics 402**  
**Spring 2019**  
**Prof. Belloni**

**Discussion Worksheet for January 30, 2019**

1. The electron in a hydrogen atom occupies the combined spin and position state

$$\Psi = R_{21}(r) \left( \sqrt{\frac{1}{3}} Y_1^0(\theta, \phi) \chi_+ + \sqrt{\frac{2}{3}} Y_1^1(\theta, \phi) \chi_- \right)$$

- a) If you measured the orbital angular momentum squared ( $L^2$ ), what values might you get, and what is the probability of each?
- b) Same for the  $z$  component of orbital angular momentum ( $L_z$ )
- c) Same for the spin angular momentum squared ( $S^2$ )
- d) Same for the  $z$  component of spin angular momentum ( $S_z$ )

2. Show that it is impossible for a spin-1/2 particle to be in a state  $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$  such that:

$$\langle S_x \rangle = \langle S_y \rangle = \langle S_z \rangle = 0.$$

Hint: start by examining  $\langle \sigma_x \rangle \sim \langle S_x \rangle$ .

Reminder: Pauli spin matrices:  $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ,  $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ ,  $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$