PHYS 402
Bose-Einstein Condensation

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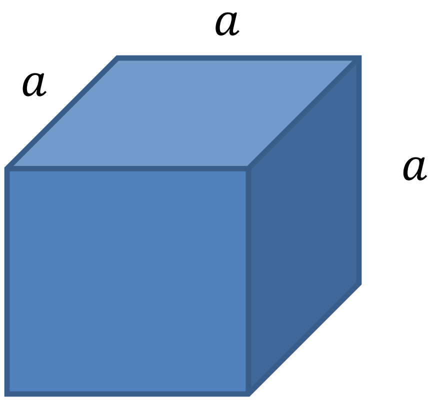
Review: ^{4}He in 3D-infinite well

- Bose-Einstein distribution

$$n_s = \frac{g_s}{\exp\left(\frac{E_s - \mu}{k_B T}\right) - 1}$$

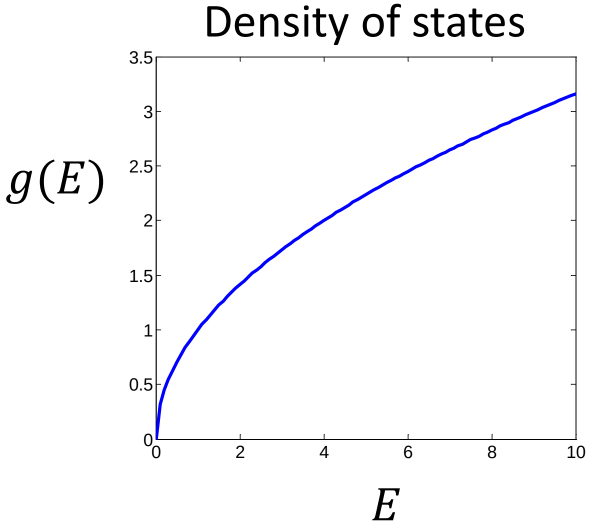


- Energy

$$E_s = \frac{\hbar^2 k^2}{2m} \qquad \vec{k} = \frac{\pi}{a}(l, m, n)$$

- Degeneracy

$$g(E) = 2\pi V \left(\frac{2m}{h^2}\right)^{3/2} E^{1/2}$$



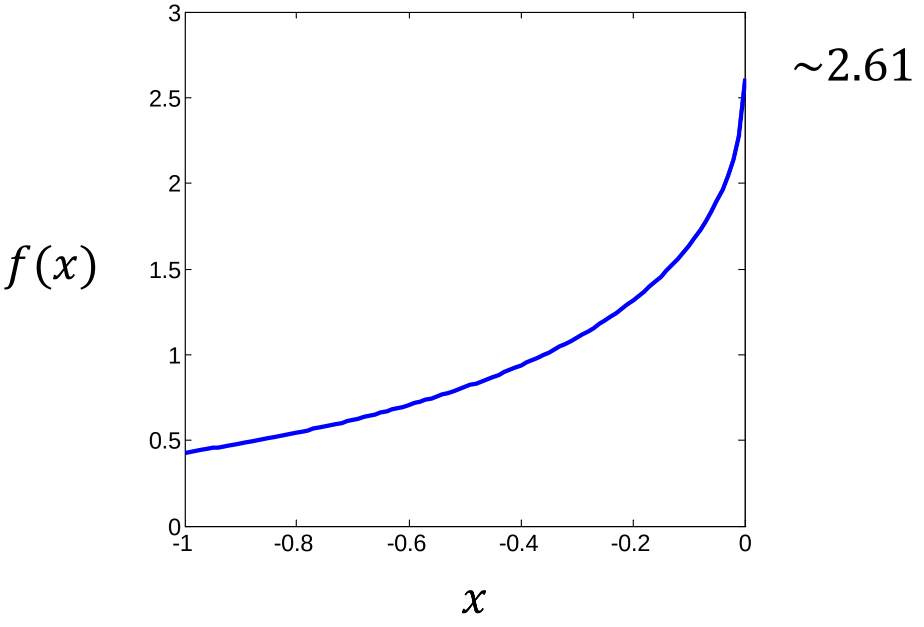
Review: ^{4}He in 3D-infinite well

- Chemical potential $\mu < 0$

$$N = \sum_{s=1}^{\infty} n_s$$

$$N = 2\pi V \left(\frac{2m}{h^2}\right)^{3/2} (k_B T)^{3/2} \Gamma(3/2) f\left(\frac{\mu}{k_B T}\right)$$

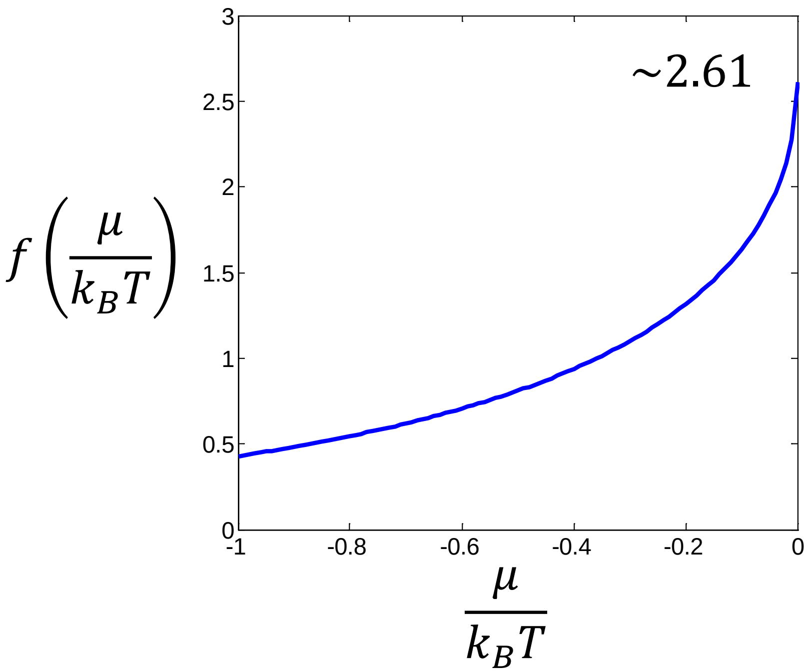
$$f(x) \equiv \sum_{p=1}^{\infty} \frac{e^{px}}{p^{3/2}}$$

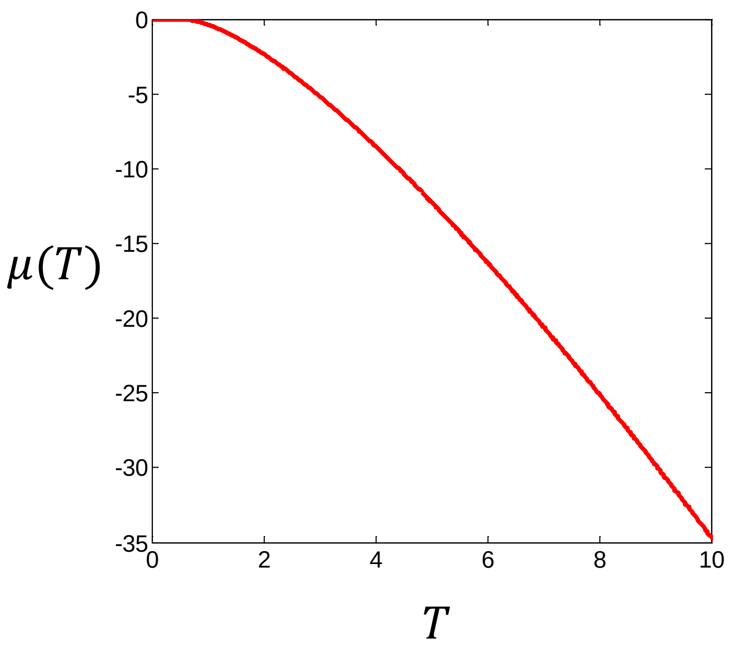


Chemical potential

$$N = 2\pi V \left(\frac{2m}{h^2}\right)^{3/2} (k_B T)^{3/2} \Gamma(3/2) f\left(\frac{\mu}{k_B T}\right)$$

$$f\left(\frac{\mu}{k_B T}\right) = \frac{N/V}{2\pi \left(\frac{2m}{h^2}\right)^{3/2} (k_B T)^{3/2} \Gamma(3/2)}$$





Crisis when $T \to 0$

$$N = 2\pi V \left(\frac{2m}{h^2}\right)^{3/2} (k_B T)^{3/2} \Gamma(3/2) f\left(\frac{\mu}{k_B T}\right)$$

Problem: Continuum approximation

$$N = \sum_{s=1}^{\infty} n_s \quad \Rightarrow \quad N = \int_0^{\infty} \frac{g(E)dE}{\exp\left(\frac{E-\mu}{k_B T}\right) - 1}$$

$$g(E) = 2\pi V \left(\frac{2m}{h^2}\right)^{3/2} E^{1/2}$$

Ground state ($E = 0$) missing!

Fixed:

$$N = \frac{1}{\exp\left(-\frac{\mu}{k_B T}\right) - 1} + 2\pi V \left(\frac{2m}{h^2}\right)^{3/2} (k_B T)^{3/2} \Gamma(3/2) f\left(\frac{\mu}{k_B T}\right)$$

Bose-Einstein condensation

Condensation temperature

when $f(x)$ is forced to be maximum ($\sim$2.61)

$$N = 2\pi V \left(\frac{2m}{h^2}\right)^{3/2} (k_B T)^{3/2} \Gamma(3/2) f\left(\frac{\mu}{k_B T}\right)$$

$$T_c = \left[\frac{N/V}{2\pi \left(\frac{2mk_B}{h^2}\right)^{\frac{3}{2}} \Gamma\left(\frac{3}{2}\right) \times 2.61}\right]^{\frac{2}{3}}$$

	^{4}He	Na (cold atom)
m	4 amu	23 amu
N/V	$2 \times 10^{28}\ \mathrm{m^{-3}}$	$10^{20}\ \mathrm{m^{-3}}$
Predicted T_c	$3.1\ \ \mathrm{K}$	$1.5\ \ \mu\mathrm{K}$
Experimental T_λ λ-transition Condensation temperature	$2.2\ \ \mathrm{K}$	$2\ \ \mu\mathrm{K}$

due to weak interaction

Bose-Einstein condensation

$$N = \frac{1}{\exp\left(-\frac{\mu}{k_B T}\right) - 1} + 2\pi V \left(\frac{2m}{h^2}\right)^{3/2} (k_B T)^{3/2} \Gamma(3/2) f\left(\frac{\mu}{k_B T}\right)$$

When $T \to 0$, a large fraction of bosons occupy the lowest quantum state.