PROBABILITY CURRENT

Using the time-dependent Schrödinger equation we can calculate the rate of change of the probability density $|\Psi|^2$:

$$\begin{aligned} |\Psi(x,t)|^2 &= \Psi^* \frac{\partial}{\partial t}\Psi + \frac{\partial}{\partial t}\Psi^*\Psi \\ &= \Psi^* \left[\frac{i\hbar}{2M}\frac{\partial^2}{\partial x^2}\Psi - \frac{i}{\hbar}V\Psi \right] + \left[-\frac{i\hbar}{2M}\frac{\partial^2}{\partial x^2}\Psi^* + \frac{i}{\hbar}V\Psi^* \right]\Psi \\ &= -\frac{\partial}{\partial x}\underbrace{\left[\frac{i\hbar}{2M}\left(\frac{\partial}{\partial x}\Psi^*\Psi - \Psi^*\frac{\partial}{\partial x}\Psi \right) \right]}_{\equiv J} \end{aligned} \tag{1}$$

The probability of finding the particle between a and b changes with time according to

$$\begin{aligned} \text{change rate of the probability of finding particle between a and b} &= \frac{\partial}{\partial t}\int_a^b dx |\Psi(x,t)|^2 \\ &= -\int_a^b dx \frac{\partial J}{\partial x} \\ &= J(a) - J(b) \\ &= \text{probability current entering through a} \\ &\quad\ \text{minus the probability leaving through b.} \end{aligned} \tag{2}$$

For a plane wave $\psi(x) = Ae^{ikx}$ moving to the right the current is

$$J = \frac{i\hbar}{2M}|A|^2(-ik - (ik)) = |A|^2\frac{\hbar k}{M} > 0. \tag{3}$$