QUANTUM PHYSICS I - FALL 2007

FINAL EXAM

Each item is worth 10 points.

I. A TWO-LEVEL SYSTEM

A system has the hamiltonian $\hat{H} = \Delta\left(|a\rangle\langle b| + |b\rangle\langle a|\right) + 2\Delta|c\rangle\langle c|$, where $\{|a\rangle, |b\rangle, |c\rangle\}$ form an orthonormal basis.

1. Find the energy eigenvalues.
2. Find the energy eigenvectors.
3. Suppose the system is described at $t = 0$ by $|t = 0\rangle = |a\rangle$. What is the state of the system at a later time t ?
4. Suppose the energy is measured at time t. What are the possible outcomes and their probabilities ?
5. Suppose that the energy is measured and the ground state energy is found. What is the probability for the system to be found at "position" "a" immediately after this energy measurement ?

II. SUDDENLY DEFORMED SQUARE WELL

A particle of mass m is in the ground state of an infinite one-dimensional square well potential ($V(x) = 0$ for $0 < x < L$ and $V(x) = \infty$ otherwise). At $t = 0$ the potential gets suddenly (that is, so fast that the wave function has no time to change) "stretched" to double its size ($V_{new}(x) = 0$ for $0 < x < 2L$ and $V_{new}(x) = \infty$ otherwise).

1. Find the position space wave function and the energy of the ground states of the old and the *new* potentials.
2. If the energy is measured immediately after $t = 0$ what is the probability for the particle to be in the ground state of the new potential ?
3. What is the value of $\langle p^2\rangle$ immediately after $t = 0$ but before the energy is measured ?

You may need: **Schrödinger equation**

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t).$$

$$\int_0^\Lambda dx\,\sin(\alpha x)\sin(\beta x) = \frac{\beta\cos(\beta\Lambda)\sin(\alpha\Lambda) - \alpha\cos(\alpha\Lambda)\sin(\beta\Lambda)}{\alpha^2 - \beta^2} \tag{1}$$

$$\int_0^\Lambda dx\,\sin^2(\alpha x) = \frac{\Lambda}{2} - \frac{\sin(2\alpha\Lambda)}{4\alpha} \tag{2}$$