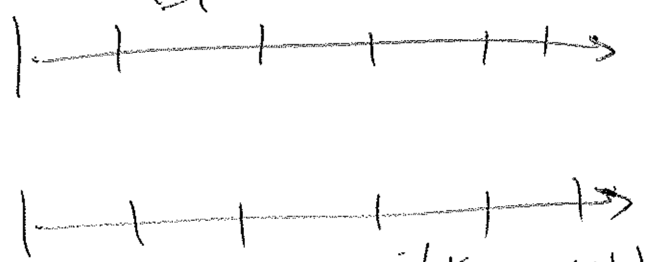
Michelson Interferometer

Two Beam Interference

$$E_1 = Ae^{i(kx_1 - \omega t)}$$



$$E_2 = Ae^{i(kx_2 - \omega t)}$$

P

waves overlap here.

$I_P \sim 4A^2 \cos^2\left(\frac{k\delta}{2}\right)$, $\delta = x_1 - x_2$ = path length difference

More generally, for any phase shift between the beams,

$I \sim 4A^2 \cos^2\left(\frac{\Delta\phi}{2}\right)$, where $\Delta\phi$ is the total phase shift between beams.

As a practical matter, we can only observe interference if the two beams are

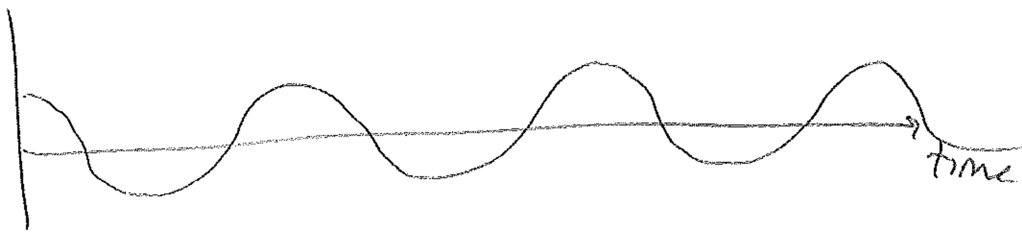
i) The same polarization.

ii) Coherent

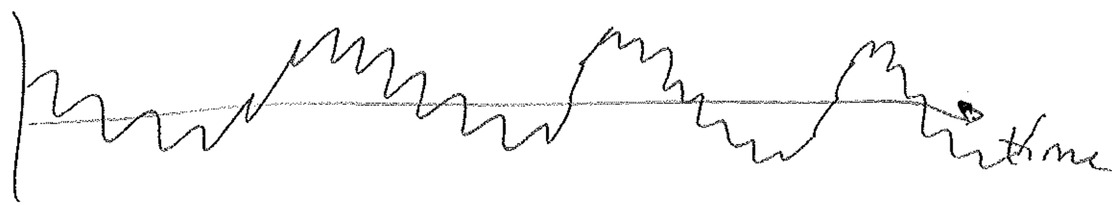
Coherence

Real EM waves are not perfect sine waves:

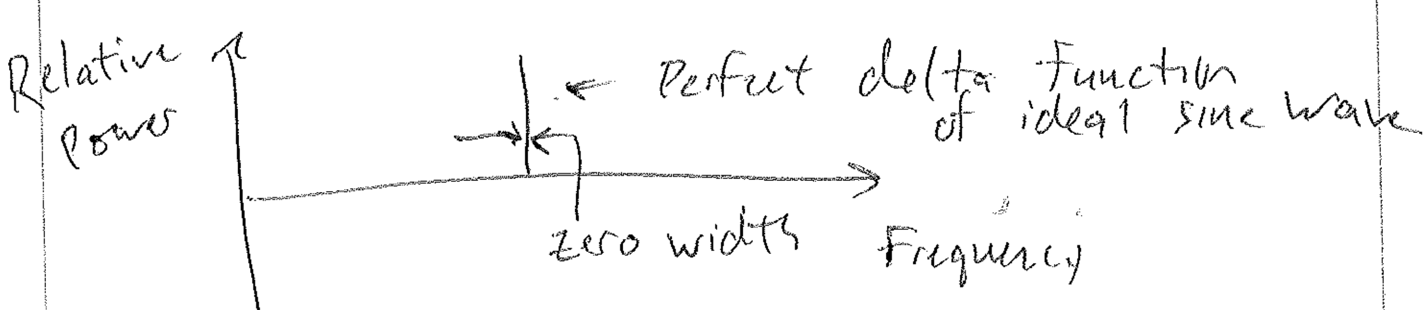
Perfect, ideal wave:

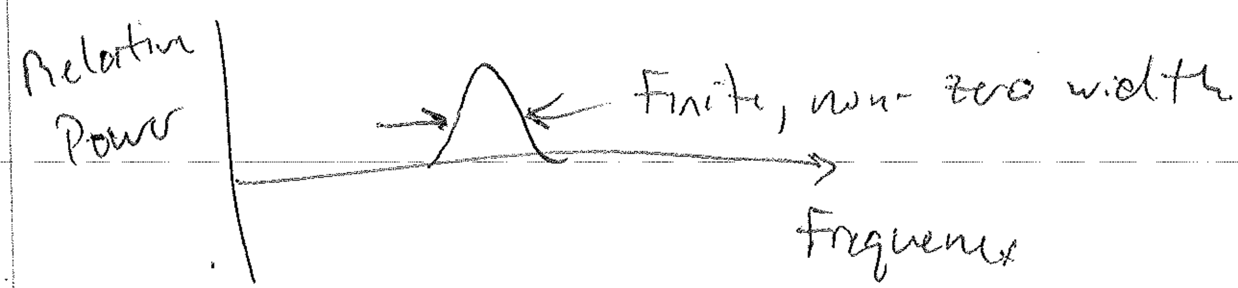


Real EM wave:



In frequency space (Fourier Transform)





Because EM waves in the real world are not perfect, they have some unpredictability.

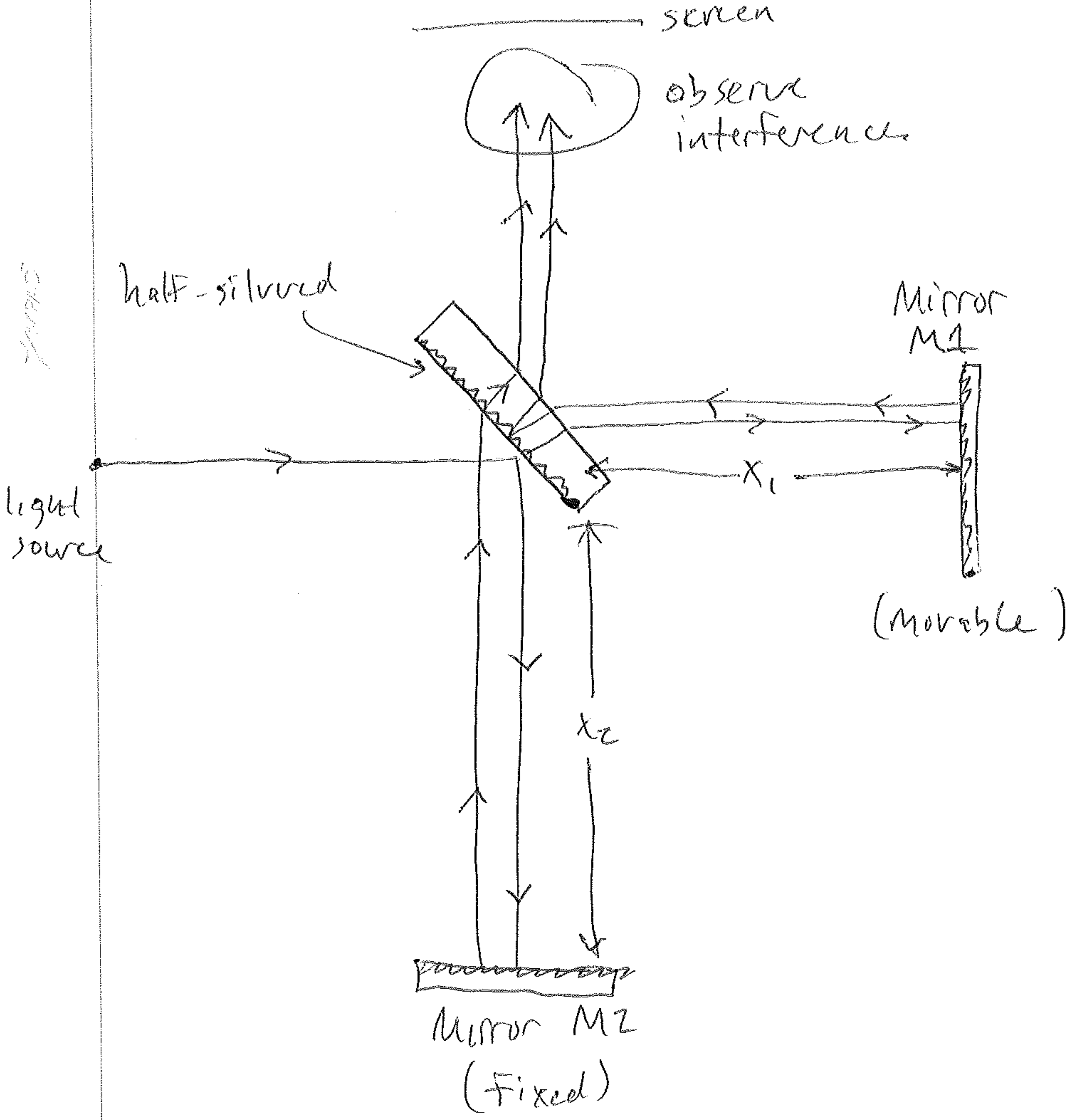
The "coherence time" is the time that the wave remains predictable.

The "coherence length" is the coherence time times the speed of light.

Light source	typical coherence time	typical coherence length
~~White light~~ Laser	$\sim (1-30)$ nano seconds	$\sim (0.2-10)$ meters.
White light	$\sim 10^{-15}$ seconds	$\sim \frac{1}{4}\,\mu$meter.

In any practical interference experiment, we force the two beams to have the same polarization and to be somewhat coherent by taking a single beam and making two copies of it. (For example, with a half-silvered beam splitting mirror.)

Michelson Interferometer



Path difference $= \delta = 2(x_2 - x_1)$

Phase difference due to path difference $=$
$$k\delta = 2k(x_2 - x_1)$$

Also, there is a phase shift of π for the beam which strikes mirror M2. This happens because this beam experiences an external reflection from the beam splitter.

So, total phase difference

$$= \Delta\phi = 2k(x_2 - x_1) + \pi$$

For constructive interference we must have $\cos^2\left(\frac{\Delta\phi}{2}\right) = 1$ or

$$\frac{\Delta\phi}{2} = m\pi, \quad m = 0, \pm 1, \pm 2, \ldots$$

"fringe order"

or $$\frac{\left(2k(x_2 - x_1) + \pi\right)}{2} = m\pi$$

$$\rightarrow k(x_2 - x_1) + \frac{\pi}{2} = m\pi$$

$k = \frac{2\pi}{\lambda}$

$$\boxed{\frac{2(x_2 - x_1)}{\lambda} = m - \frac{1}{2}}$$ for constructive interference

In practice we typically use the interferometer by moving mirror M2. Then x_2 changes, and on the screen we count the fringes which pass by (the constructive interference maxima.)

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Letting X_1 change (ΔX_1), and letting m change (Δm) we have

$$\boxed{\frac{2\,\Delta X_1}{\lambda} = \Delta m}$$

move mirror M1 by distance ΔX_1

count to number of fringes which pass by

This allows us to measure the wavelength λ by measuring Δm and ΔX_1.

Sodium Lamp

Two closely spaced wavelengths: λ, λ'

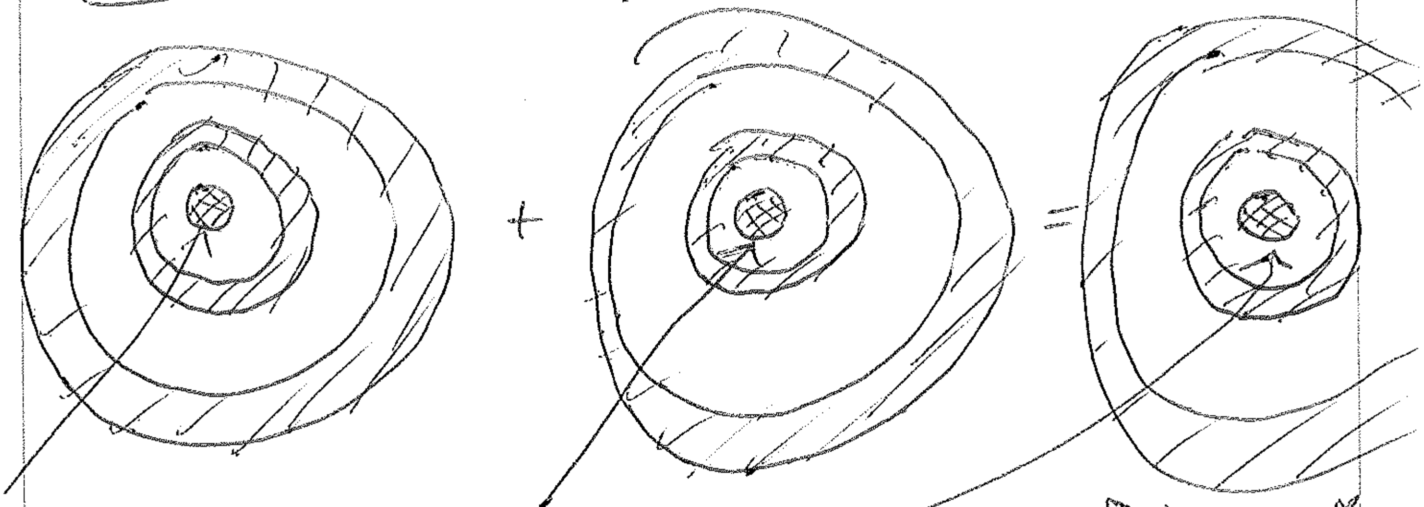
Each makes its own fringe pattern.

These fringe patterns are seen on top of each other on the screen - our eye cannot observe them individually.

Sometimes both patterns reinforce each other - they have bright locations and dark locations in the same place:

This is "high fringe visibility":

Fringe pattern for λ | Fringe pattern for λ' | what we see:



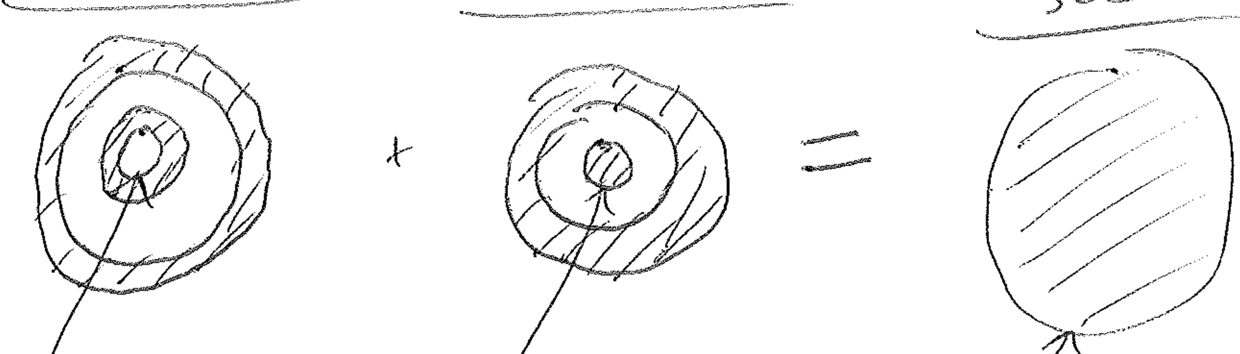
bright spot at center

fringes are visible

Low fringe visibility:

When the two patterns are opposite:

λ | λ' | what we see

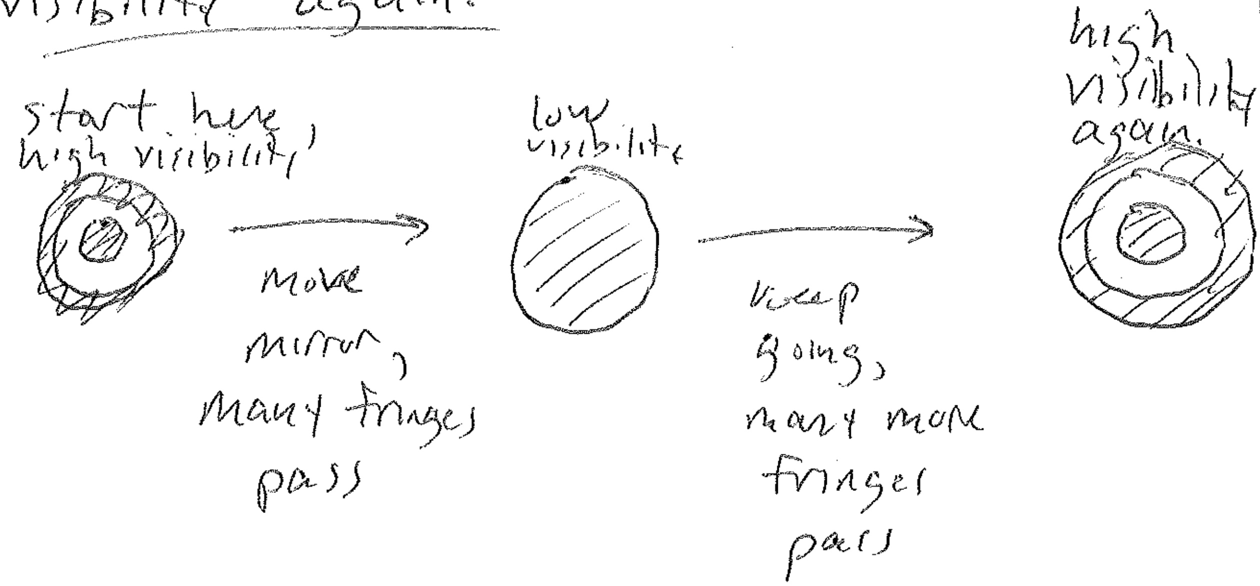


dark spot

bright spot

a wash, fringes not visible

Suppose we start with high fringe visibility, then we start moving mirror M1. We watch many fringes go by. Eventually we are seeing low fringe visibility, we continue to move the mirror, and we see many more fringes pass by, and eventually we reach high fringe visibility again.



It must be true that

$$\begin{pmatrix}\text{the number} \\ \text{of fringes} \\ \text{for } \lambda\end{pmatrix} = \begin{pmatrix}\text{the number} \\ \text{of fringes} \\ \text{for } \lambda'\end{pmatrix} + 1$$

because we passed through low visibility and returned to high

Therefore, if ΔX_1 is the total distance that we moved the mirror, then

$$\frac{2\Delta X_1}{\lambda} = \frac{2\Delta X_1}{\lambda'} + 1$$

or $$\boxed{\lambda - \lambda' = \frac{\lambda\lambda'}{2\Delta X_1}}$$

This tells us the difference in the two wavelengths.

We can simplify a little. λ and λ' are almost equal, $\lambda \approx \lambda'$, so on the right side we can write

$$\lambda - \lambda' \approx \frac{\lambda(\lambda)}{2\Delta X_1} = \frac{\lambda^2}{2\Delta X_1}$$

or $$\boxed{\Delta\lambda \approx \frac{\lambda^2}{2\Delta X_1}}$$

The difference in the two wavelengths.

To use this expression, we must know λ. How can we measure λ? By counting fringes, just like with the laser.

Strictly speaking, by counting fringes we measure the average of λ & λ', call it

$$\boxed{\bar{\lambda} = \tfrac{1}{2}(\lambda + \lambda')}$$

Then we can say

$$\boxed{\Delta\lambda \approx \frac{\bar{\lambda}^2}{2\Delta x_1}}$$

since $\bar{\lambda} \approx \lambda \approx \lambda'$.