## Phys 375 HW 1 Fall 2009 6 Problems Due 14 / 15 September, 2009

1. **Error analysis**. For the following list of data, calculate ("by hand" – i.e. use a calculator and show your work) the mean, mode, median, standard deviation, variance, and standard deviation of the mean. Please do the analysis "by hand" – just this once.

{7.127, 7.125, 7.041, 6.963, 7.125, 6.820, 7.027, 6.843, 7.067, 7.084}

a) mean = (7.127 + 7.125 + 7.041 + 6.963 + 7.125 + 6.820 + 7.027 + 6.843 + 7.067 + 7.084)/10 = 7.022

b) mode – 7.125 since it occurs twice

c) median - arrange the above list in order. Half (5) of the data points should be less than the median and 5 should be greater. Thus we take the mean of 7.041 and 7.067, finding 7.054.

d) std. deviation - 
$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - x)^2 = 0.1148/9 = 0.0128 \text{ so } \sigma = 0.113$$
  
e) std. deviation of mean -  $\frac{\sigma}{\sqrt{N}} = 0.036$ 

2. Error propagation. You are trying to determine the acceleration due to gravity g by measuring the period of a pendulum, T, of length L using the relation  $T = 2\pi \sqrt{\frac{L}{g}}$ .

The summary of measured data is  $T = 3.818 \pm 0.009$  sec, and  $L = 361.58 \pm 0.40$  cm. By propagating errors, determine the best value and uncertainty in g. If you could go back and revise the experiment, which quantity would you want to measure more precisely?

Rearrange the above equation to give (1)  $g = 4\pi^2 \frac{L}{T^2}$ . Using the error propagation formula we find (2)  $\frac{\sigma_g}{g} = \sqrt{\left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{\sigma_{T^2}}{T^2}\right)^2} = \sqrt{\left(\frac{\sigma_L}{L}\right)^2 + \left(2\frac{\sigma_T}{T}\right)^2}$ . The best value of g is given by (1)  $g = 4\pi^2 \frac{361.68cm}{(3.818 \text{ sec})^2} = 979.2 \text{ cm/s}^2$ . Plugging into (2) we find

$$\frac{\sigma_g}{g} = \sqrt{\left(\frac{0.40}{361.8}\right)^2 + \left(2\frac{0.009}{3.818}\right)^2} = 0.00484 \text{ and } \sigma_g = 4.7 \text{ cm/s}^2.$$
 In the previous

formula the ratio of the length uncertainty to the length is small so the error is dominated by the uncertainty in the period.

3. Pedrotti<sup>3</sup>, 3<sup>rd</sup> edition, problem 1-3.

The energy of a photon is given by  $E = h v = hc/\lambda$ . Thus at  $\lambda = 380$  nm and 770 nm,  $E = (6.63x10^{-34} J*s)(3x10^8 m/s)/(380x10^{-9} m) = 5.23x10^{-19} J$  and  $(6.63x10^{-34} J*s)(3x10^8 m/s)/(770x10^{-9} m) = 2.58x10^{-19} J$ . Since  $1eV = 1.6x10^{-19} J$  the energies are 3.27 eV and 1.61 eV respectively.

4. Pedrotti<sup>3</sup>, 3<sup>rd</sup> edition, problem 1-15.

(a) 
$$t = D_l/c = (90x10^3)/3.0x10^8 s = 3.0 x 10^4 s$$
  
(b)  $D_s = v_s t = (340)(3.0x10^4 m) = 0.10 m$ 

5. An electromagnetic wave is specified (in SI units) by the following function:

$$\vec{E} = (-6\hat{i} + 3\sqrt{5}\hat{j})(10^4 V/m) \exp[i\{\frac{1}{3}(\sqrt{5}x + 2y)\pi \times 10^7 - 9.42 \times 10^{15}t\}]$$

Find (a) the direction along which the electric field oscillates,

The field oscillates in  $(-6\hat{i} + 3\sqrt{5}\hat{j})$  direction. Normalizing this gives the unit vector  $(-\frac{2}{3}\hat{i} + \frac{\sqrt{5}}{3}\hat{j})$ . One could also find an angle direction with respect to the x-axis using  $\tan \theta = \sqrt{5}/2 = 48.2+90 = 138.2$  degrees

(b) the scalar value of amplitude of electric field,

Take the dot product of the amplitude and then the square root.  $\sqrt{36+45} \, x \, 10^4 \, V/m = 9 \, x \, 10^4 \, V/m$ 

(c) the direction of propagation of the wave,

Since the exponential is  $\vec{k} \cdot \vec{r} - \omega t$ , the wave travels in the direction of  $\vec{k}$ .

(d) the propagation number and wavelength,

The dot product of  $\vec{k}$  and  $\vec{r}$  in the exponential is  $\{\frac{1}{3}(\sqrt{5}x+2y)\pi \times 10^7 \text{ which}$ implies  $\vec{k} = \sqrt{5}\hat{i} + 2\hat{j}(\pi/3)10^7$ . Then  $\vec{k} \cdot \vec{k} = (\pi x 10^7)^2$  so  $k = \pi x 10^7 m^{-1}$ . Finally,  $\lambda = \frac{2\pi}{k} = 200 nm$ .

(e) the frequency and angular frequency, and

$$\omega = 9.42 x 10^{15}$$
 rad/s and  $v = \omega/2\pi = 1.5 x 10^{15}$  Hz

(f) the speed.

$$v = v\lambda = 3x10^8 m/s$$
, the speed of light.

6. An underwater swimmer shines a beam of light up toward the surface. It strikes the air-water interface at  $35^{\circ}$ . At what angle will it emerge into the air?

The index of refraction of water is 1.33. Using Snell's Law we get 1.33 sin  $35 = 1.00 \sin \theta_t$ . Solving,  $\theta_t = 50$  degrees.