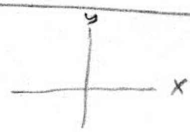
79b

$$V_x(x,y)\,dx + V_y(x+dx,y)\,dy - V_x(x,y+dy)\,dx - V_y(x,y)\,dy$$
$$= V_x(x,y)\,dx + V_y(x,y)\,dy + \frac{\partial V_y}{\partial x}(x,y)\,dx\,dy - V_x(x,y)\,dx - \frac{\partial V_x}{\partial y}(x,y)\,dx\,dy - V_y(x,y)\,dy$$
$$= (\partial_x V_y - \partial_y V_x)\,dx\,dy$$

83a

$$\boxed{\vec{V} = f(y)\hat{x}}$$



the flow is only in the x-direction. No matter where the particle starts, all it does is move to the right (so it can never return to it's original position.

83b

$$\nabla \times \vec{v} = -\frac{\partial f}{\partial y}\hat{z}$$

83c

$$\boxed{\vec{V} = V_0 e^{-\frac{y^2}{c^2}}\hat{x}}$$

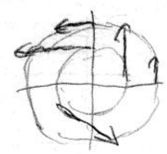
$$\nabla \times \vec{V} = -V_0 \frac{\partial e^{-\frac{y^2}{c^2}}}{\partial y}\hat{z} = V_0\left(\frac{2y}{c^2}\right)e^{-\frac{y^2}{c^2}}\hat{z}$$

and it is positive for the upper paddle wheel [since $y>0$] and negative for the lower paddle wheel [since $y<0$]

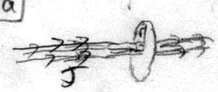
84d

$$\boxed{\vec{V} = \frac{A}{r}\hat{\phi}}$$

$$\nabla \times \vec{v} = \left(\frac{\partial v}{\partial r} + \frac{v}{r}\right)\hat{z} = \left[-\frac{A}{r^2} + \frac{A}{r^2}\right]\hat{z} = 0$$



85a



due to symmetry,
$\vec{B} = \vec{B}(r)$; $\vec{B}$ is a function of r, not ϕ or z

$$\boxed{\vec{B} = f(r)\hat{\phi}}$$

Away from wire $\nabla \times \vec{B} = 0$

$$\frac{\partial}{\partial r}(rf) = 0$$

$$\boxed{f(r) = \frac{A}{r}}$$

85b

We didn't use anything involving the magnitude of $\vec{J}$, only the direction (because in part a, we only looked in the region away from wire where $\nabla \times \vec{B} = 0$, and so we didn't use any info involving the magnitude of $\vec{J}$)

89a

$\boxed{\vec{v} = (x+y+z)\hat{z}}$

$$\oint_S \vec{v} \cdot d\vec{s} = \int ds\,(x+y+z)\cos\theta$$

$$= R^3 \int_0^{2\pi}\int_0^{\pi} [\sin\theta\cos\phi + \sin\theta\sin\phi + \cos\theta] \sin\theta\cos\theta \, d\phi\, d\theta$$

$$= R^3 \int_0^{\pi} \sin\theta\cos^2\theta \, d\theta \, (2\pi)$$

$$= -2\pi R^3 \int u^2$$

$$= -2\pi R^3 \frac{(\cos\theta)^3}{3} \Big|_0^{\pi}$$

$$= \frac{4\pi R^3}{3}$$

89b

$\boxed{\nabla \cdot \vec{v} = 1}$

$$\int_V (\nabla \cdot \vec{v})\, dV = \int_V (1)\, dV = \frac{4}{3}\pi R^3$$

another 89a

$\boxed{\nabla \cdot \vec{g} = -4\pi G\rho \quad \text{eq. 6.33}}$

$$\int_V (\nabla \cdot \vec{g})\, dV = -4\pi G \int_V \rho\, dV$$

$$\oint_S \vec{g} \cdot d\vec{s} = -4\pi G M$$

another 89b

$$\oint_S \vec{g} \cdot d\vec{s} = -4\pi G M$$

$$4\pi r^2 g = -4\pi G M$$

$$g(r) = \frac{-GM}{r^2}$$

$$\vec{g}(r) = \frac{-GM}{r^2}\hat{r}$$

90c

$$\nabla\cdot\vec{g} = -4\pi G\rho$$

$$\int_V \nabla\cdot\vec{g} = -\int_V 4\pi G\rho$$

$$\oint_S \vec{g}\cdot d\vec{s} = -\int_V 4\pi G\rho$$

$$4\pi r^2 g = -4\pi G\int_V \rho$$

$$g = -\frac{G}{r^2} M\frac{r^3}{R^3}$$

$$\vec{g}(\vec{r}) = -\frac{GMr}{R^3}\hat{r}$$

91d

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint_S \vec{E}\cdot d\vec{s} = \int_V \frac{\rho}{\varepsilon_0}$$

$$4\pi r^2 E = 0$$

W.1

$$\nabla\times\vec{v} = \nabla\times[\vec{\omega}\times\vec{r}]$$

$$= \nabla\times(\omega_i x_j \hat{e}_k \varepsilon_{ijk})$$

$$= \partial_l \omega_i x_j \varepsilon_{lkm}\varepsilon_{ijk}\hat{e}_m$$

$$= \omega_i \varepsilon_{jkm}\varepsilon_{ijk}\hat{e}_m$$

$$= 2\delta_{mi}\omega_i\hat{e}_m$$

$$= 2\omega_i\hat{e}_i$$

$$= 2\vec{\omega}$$