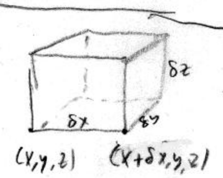
51e



On the left pane, there's a force, $p(x,y,z)\delta y\delta z$ to the right

On the right pane, there's a force $p(x+\delta x,y,z)\delta y\delta z$ to the left

52f

$$\boxed{f_x \simeq -\delta x\frac{\partial p}{\partial x}\delta y\delta z\,\hat{x} = -\frac{\partial p}{\partial x}\delta V\,\hat{x}}$$

52g

$$\vec{f} = f_x\hat{x} + f_y\hat{y} + f_z\hat{z} = -\left[\frac{\partial p}{\partial x}\hat{x} + \frac{\partial p}{\partial y}\hat{y} + \frac{\partial p}{\partial z}\hat{z}\right]\delta V = -\nabla p\,\delta V$$

54b

$$\oint(\nabla f\cdot d\vec{r}) = f_A - f_A = 0$$

54c

$$\frac{df}{ds}(\vec{r}) = \lim_{\delta s\to 0}\frac{f(\vec{r}+\hat{n}\delta s)-f(\vec{r})}{\delta s} = \lim_{\delta s\to 0}\frac{\nabla f\cdot(\hat{n}\delta s)}{\delta s} = \nabla f\cdot\hat{n}$$

56a

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{d}{dt}\left(\frac{1}{2}m\vec{v}\cdot\vec{v}\right) = m\vec{v}\cdot\frac{d\vec{v}}{dt}$$

56b, 56c

$$\frac{dV}{dt}(\vec{r}) = \lim_{\delta t\to 0}\frac{V(\vec{r}(t+\delta t)) - V(\vec{r}(t))}{\delta t} = \lim_{\delta t\to 0}\frac{\nabla V\cdot\delta\vec{r}}{\delta t} = \vec{v}\cdot\nabla V$$

56d

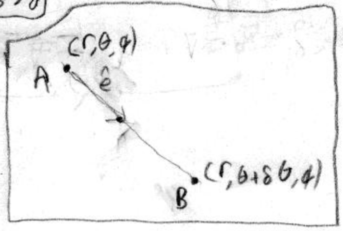
$$\frac{1}{2}mv^2 + V(\vec{r}) = E$$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) + \frac{dV}{dt} = 0$$

$$m\vec{v}\cdot\frac{d\vec{v}}{dt} + \vec{v}\cdot\nabla V = 0$$

$$\vec{v}\cdot\left[m\frac{d\vec{v}}{dt} + \nabla V\right] = 0$$

63d



$$\boxed{\nabla f \cdot \hat{e} = \frac{\delta f}{\delta s} = \frac{\delta f}{r\,\delta\theta} = \frac{1}{r}\frac{\partial f}{\partial \theta}}$$

Since $\frac{\delta f}{\delta s} = \lim \frac{f(\vec{r}+\hat{e}\delta s) - f(\vec{r})}{\delta s} = \lim \frac{f(B) - f(A)}{\delta s} = \lim \frac{f(r,\theta+\delta\theta,\phi) - f(r,\theta,\phi)}{r\,\delta\theta}$

63f

$$\nabla f \cdot \hat{e} = \frac{\delta f}{\delta s} = \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

63g

$$\left[\frac{\partial}{\partial r}\right] = \left[\frac{1}{r}\frac{\partial}{\partial \theta}\right] = \left[\frac{1}{r\sin\theta}\right]$$

63h

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$