6.b

No.
The right hand side has dimensions involving the mass.

7d

right hand side has dimension

$$\left[\frac{ML^2}{T}\right]\left[\frac{1}{T^2}\right]=\left[\frac{ML^2}{T^3}\right]$$

which is not the dimensions of energy

6d

$$\hbar \sim \left[\frac{1}{T}\right][M][L^2]$$

$$\hbar \sim \left[\frac{ML^2}{T}\right]$$

11a

$\rho \sim \left[\frac{M}{L^3}\right]$ $v \sim \left[\frac{L}{T}\right]$ $S \sim [L^2]$

11b

variables: $\rho, v, W, S$

physical dimensions: $[L], [M], [T]$

11c

$$\boxed{\left[\frac{ML}{T^2}\right]\left[\frac{M^\alpha}{L^{3\alpha}}\right]\left[\frac{L^\beta}{T^\beta}\right][L^{2\gamma}]\sim[1]}$$

$$\boxed{\left[L^{1+2\gamma-3\alpha+\beta}\right]\sim[1]} \Rightarrow \boxed{1-3\alpha+\beta+2\gamma=0}$$

$$\boxed{[M^{1+\alpha}]\sim[1]} \Rightarrow \boxed{1+\alpha=0}$$

$$\boxed{[T^{-2-\beta}]\sim[1]} \Rightarrow \boxed{2+\beta=0}$$

11d

$$\boxed{\begin{aligned}\alpha&=-1\\ \beta&=-2\end{aligned}}$$

$$\boxed{\begin{aligned}1-3(-1)+(-2)+2\gamma&=0\\ 2\gamma&=-2\\ \gamma&=-1\end{aligned}}$$

**13d**

$$\boxed{W = g\rho_f V}$$

$$g\rho_f V^{\frac{1}{3}} = C_L \rho V^2$$

$$V = \left(\frac{C_L \rho}{g\rho_f}\right)^3 V^6$$

$$W = g\rho_f \left(\frac{C_L \rho}{g\rho_f}\right)^3 V^6$$

$$= \frac{C_L^3 \rho^3}{g^2 \rho_f^2} V^6$$

**15a**

$$\boxed{\frac{\partial p}{\partial x} \sim \left[\frac{1}{L}\right]\left[\frac{M}{T^2 L^2}\right] = \left[\frac{M}{T^2 L^2}\right]}$$

$$\boxed{\mu = \frac{\tau}{\frac{\partial v}{\partial z}}} \Rightarrow \boxed{\mu \sim \left[\frac{M}{T^2 L}\right] \frac{1}{\left[\frac{1}{L}\right]\left[\frac{L}{T}\right]} = \left[\frac{M}{TL}\right]}$$

$$\boxed{R \sim [L]}$$

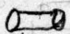
$$\boxed{\Phi \sim \left[\frac{L^3}{T}\right]}$$

**15b**

$$\left[\frac{M}{T^2 L^3}\right]\left[\frac{M^\alpha}{T^\alpha L^{2\alpha}}\right]\left[L^\beta\right]\left[\frac{L^{3\gamma}}{T^\gamma}\right] \sim [1] \Rightarrow \begin{matrix} 1+\alpha=0 \\ 2+\alpha+\gamma=0 \\ 3\gamma+\beta-3-2\alpha=0 \end{matrix} \Rightarrow \begin{matrix} \alpha=-1 \\ \gamma=-1 \\ \beta=4 \end{matrix}$$

$$\text{const.} \frac{\partial p}{\partial x} R^4 = \phi\mu \Rightarrow \phi = \text{const.} \frac{\frac{\partial p}{\partial x}}{\mu} R^4$$

15c

$\Phi$ is defined as the volume of flow per unit time
Volume goes like $R^2$ 
But $\Phi$ is also proportional to the driving force which is proportional to $\frac{\partial P}{\partial x} R^2$ which is where the other $R^2$ comes from

23d

$$\delta r \approx \frac{\partial r}{\partial V} \delta V$$
$$= \frac{1}{3}\left(\frac{3V}{4\pi}\right)^{-\frac{2}{3}}\left(\frac{3}{4\pi}\right)\delta V$$
$$= \frac{1}{3} r \left(\frac{4\pi}{3V}\right)\left(\frac{3}{4\pi}\right)\delta V$$
$$= \frac{1}{3} r \frac{\delta V}{V}$$

25a

$v \uparrow o$

$$\boxed{t = \frac{2V}{g}}$$

$$\boxed{\frac{1}{2}v^2 = gh \qquad h = \frac{v^2}{2g}}$$

25b

$$V_n^2 = (1-\gamma) V_{n-1}^2$$
$$V_n = \sqrt{1-\gamma}\, V_{n-1}$$

25c

$$\boxed{t_n = \frac{2}{g} V_n = \sqrt{1-\gamma}\, t_{n-1}}$$

$$\boxed{t_0 = \frac{2}{g}\sqrt{2gH} = \sqrt{\frac{8H}{g}}}$$

$$\boxed{t_n = (1-\gamma)^{\frac{n}{2}} \sqrt{\frac{8H}{g}}}$$

25d

$$T_N = \sqrt{\frac{8H}{g}} \sum (1-r)^{\frac{n}{2}}$$

$$= \sqrt{\frac{8H}{g}} \sum x^n \text{ ; where } x = \sqrt{1-r}$$

$$T_\infty = \sqrt{\frac{8H}{g}} \frac{1}{1-x}$$

$$= \sqrt{\frac{8H}{g}} \frac{1}{1-\sqrt{1-r}}$$

25e

$$(1-r)^{\frac{1}{2}} = 1 - \frac{1}{2} r + \dots$$

$$T_\infty \approx \sqrt{\frac{8H}{g}} \frac{1}{1-(1-\frac{r}{2})}$$

$$= \sqrt{\frac{8H}{g}} \frac{2}{g}$$

25f

$$d_n = 2h_n$$

$$= \frac{\cancel{2} V_n^2}{\cancel{2} g}$$

$$= \frac{V_n^2}{g}$$

$$S = \sum d_n$$

$$= \frac{1}{g} \sum V_n^2$$

$$V_0^2 = 2gH$$

$$V_n^2 = (1-r) V_{n-1}^2 = (1-r)^n V_0^2 = (1-r)^n \, 2gH$$

$$S = \frac{1}{g}(2gH) \sum (1-r)^n = \frac{2H}{1-(1-r)} = \frac{2H}{r}$$

# HW #2



**28a**

$$B = A R_R$$
$$T = A T_R$$
$$R = R_L + B T_L$$

**28b**

$$A = T_L + A R_R R_L$$
$$\boxed{A = \frac{T_L}{1 - R_L R_R}}$$

$$\boxed{B = \frac{T_L R_R}{1 - R_L R_R}}$$

**28c**

$$\boxed{(1 - R_L R_R)^{-1} = 1 + R_L R_R + (R_L R_R)^2 + \cdots}$$

$$A = T_L + T_L R_R R_L + T_L R_R R_L R_R R_L + \cdots$$
$$B = T_L R_R + T_L R_R R_L R_R + \cdots$$

**29d**

$$R = R_L + \frac{T_L^2 R_R}{(1 - R_L R_R)}$$

$$T = \frac{T_L T_R}{1 - R_L R_R}$$

**32b**

$$z = r\cos\theta$$
$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$

**33d**

$$\frac{\partial \vec{r}}{\partial x} = \frac{\partial}{\partial x}(x\hat{x} + y\hat{y} + z\hat{z}) = \hat{x}$$

**33e**

| | |
|---|---|
| $\frac{\partial \vec{r}}{\partial r} = \frac{\partial x}{\partial r}\hat{x} + \frac{\partial y}{\partial r}\hat{y} + \frac{\partial z}{\partial r}\hat{z} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$ | $\hat{r} = \frac{\partial \vec{r}}{\partial r}$ |
| $\frac{\partial \vec{r}}{\partial \theta} = r[\cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}]$ | $\hat{\theta} = \frac{1}{r}\frac{\partial \vec{r}}{\partial \theta}$ |
| $\frac{\partial \vec{r}}{\partial \phi} = r\sin\theta[-\sin\phi\,\hat{x} + \cos\phi\,\hat{y}]$ | $\hat{\phi} = \frac{1}{r\sin\theta}\frac{\partial \vec{r}}{\partial \phi}$ |

**34f**

$$(\hat{r}\cdot\hat{r}) = (\hat{\theta}\cdot\hat{\theta}) = (\hat{\phi}\cdot\hat{\phi}) = 1$$
$$(\hat{r}\cdot\hat{\theta}) = (\hat{r}\cdot\hat{\phi}) = (\hat{\theta}\cdot\hat{\phi}) = 0$$

**34g**

$$\hat{r}\times\hat{\theta} = \hat{\phi}$$
$$\hat{\theta}\times\hat{\phi} = \hat{r}$$
$$\hat{\phi}\times\hat{r} = \hat{\theta}$$

**34h**

| | |
|---|---|
| $\frac{\partial\hat{r}}{\partial\theta} = \hat{\theta}$ | $\frac{\partial\hat{r}}{\partial\phi} = \sin\theta\,\hat{\phi}$ |
| $\frac{\partial\hat{\theta}}{\partial\theta} = -\hat{r}$ | $\frac{\partial\hat{\theta}}{\partial\phi} = \cos\theta\,\hat{\phi}$ |
| $\frac{\partial\hat{\phi}}{\partial\theta} = 0$ | $\frac{\partial\hat{\phi}}{\partial\phi} = -\sin\theta\,\hat{r} - \cos\theta\,\hat{\theta}$ |

**36d**

$$MM^T = M^TM = 1$$

**36e**

$$\begin{pmatrix} u_r \\ u_\theta \\ u_\phi \end{pmatrix} = M\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

**37a**

$$\frac{d\hat{r}}{dt} = \frac{d\theta}{dt}\hat{\theta} + \frac{d\phi}{dt}\sin\theta\,\hat{\phi}$$

$$\frac{d\hat{\theta}}{dt} = -\frac{d\theta}{dt}\hat{r} + \cos\theta\frac{d\phi}{dt}\hat{\phi}$$

$$\frac{d\hat{\phi}}{dt} = -\frac{d\phi}{dt}\sin\theta\,\hat{r} - \frac{d\phi}{dt}\cos\theta\,\hat{\theta}$$

**38b**

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{d(r\hat{r})}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\theta}{dt}\hat{\theta} + r\sin\theta\frac{d\phi}{dt}\hat{\phi}$$

**38d**

$$\vec{a} = \frac{d\vec{V}}{dt}$$

$$= \frac{dV_r}{dt}\hat{r} + V_r\left(\frac{d\theta}{dt}\hat{\theta} + \sin\theta\frac{d\phi}{dt}\hat{\phi}\right) + \frac{dV_\theta}{dt}\hat{\theta} + V_\theta\left(-\frac{d\theta}{dt}\hat{r} + \cos\theta\frac{d\phi}{dt}\hat{\phi}\right)$$

$$+ \frac{dV_\phi}{dt}\hat{\phi} + V_\phi\left(-\sin\theta\frac{d\phi}{dt}\hat{r} - \cos\theta\frac{d\phi}{dt}\hat{\theta}\right)$$

$$= \left(\frac{dV_r}{dt} - \frac{d\theta}{dt}V_\theta - \sin\theta\frac{d\phi}{dt}V_\phi\right)\hat{r} + \left(\frac{dV_\theta}{dt} + \frac{d\theta}{dt}V_r - \cos\theta\frac{d\phi}{dt}V_\phi\right)\hat{\theta}$$

$$+ \left(\frac{dV_\phi}{dt} + \cos\theta\frac{d\phi}{dt}V_\theta + \sin\theta\frac{d\phi}{dt}V_r\right)\hat{\phi}$$

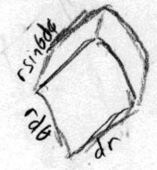
**39e**

$$\boxed{a_r = \frac{dV_r}{dt} - \frac{V_\theta^2 + V_\phi^2}{r}}$$

$$\boxed{a_\theta = \frac{dV_\theta}{dt} + \frac{V_\theta V_r}{r} - \frac{V_\phi^2}{r\tan\theta}}$$

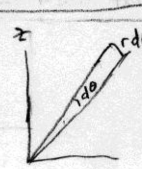
$$\boxed{a_\phi = \frac{dV_\phi}{dt} + \frac{V_\phi V_r}{r} + \frac{V_\theta V_\phi}{r\tan\theta}}$$

**41d**

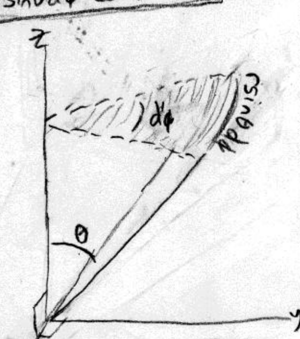



Volume $= r^2\sin\theta\, d\theta\, d\phi\, dr$

to see where the $rd\theta$ comes from




to see where the $r\sin\theta d\phi$ comes from



43g


$$dS = r^2 \sin\theta\, d\theta\, d\phi$$

43h

$$\int_0^R \int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta\, d\theta\, d\phi\, dr = (2\pi)\left(\frac{R^3}{3}\right)\left(-\cos\theta\right)\Big|_0^{\pi} = \frac{4\pi R^3}{3}$$