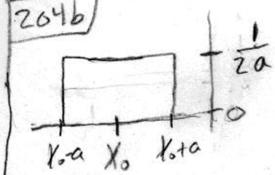
HW#7

(1)

204a

$$\int_{-\infty}^{\infty} B_a(x)\,dx = 2\int_0^a \frac{1}{2a}\,dx = \frac{1}{a}x\Big|_0^a = 1$$

204b



$$\int_{-\infty}^{\infty} B_a(x-x_0)f(x)\,dx = \frac{1}{2a}\int_{x_0-a}^{x_0+a} f(x)\,dx$$

205c

it's equation 3.17

205d

for odd n — goes to 0 because of symmetry

$$\int_{x_0-a}^{x_0+a} (x-x_0)^n\,dx = \int_{-a}^{a} x^n = \frac{x^{n+1}}{n+1}\Big|_{-a}^{a} = \frac{2a^{n+1}}{n+1}$$

205e

$$\int_{-\infty}^{\infty} B_a(x-x_0)f(x)\,dx = \frac{1}{2a}\sum_n \frac{1}{n!}\frac{d^nf}{dx^n}(x_0)\int_{x_0-a}^{x_0+a}(x-x_0)^n\,dx$$

$$= \frac{1}{2a}\sum_{n\ \text{even}} \frac{1}{n!}\frac{d^nf}{dx^n}(x_0)\,\frac{2a^{n+1}}{n+1}$$

$$= \sum_{\text{even}} \frac{1}{(n+1)!}\frac{d^nf}{dx^n}(x_0)\,a^n$$

206a

use eq. 14.11

207b

$$\int_{-\infty}^{\infty} g_a(x)\,dx = \frac{1}{a\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-\frac{x^2}{a^2}}\,dx = \frac{1}{a\sqrt{\pi}}\left[a\sqrt{\pi}\right] = 1$$

207c

$$\int_{-\infty}^{\infty} \delta(c(x-x_0))\,f(x)\,dx \; ; \quad \begin{matrix} y = cx \\ dy = c\,dx \end{matrix}$$

$$= \int_{-\infty}^{\infty} \delta(y - cx_0)\, f\left(\tfrac{y}{c}\right) \frac{dy}{c}$$

207d

$$= \frac{1}{c}\int_{-\infty}^{\infty} \delta(y-cx_0)\, f\left(\tfrac{y}{c}\right) dy$$

$$= \frac{1}{c} f\left(\frac{cx_0}{c}\right)$$

$$= \frac{1}{c} f(x_0)$$

207e

$$\int_{x=-\infty}^{x=\infty} \delta(c(x-x_0))\, f(x)\,dx$$

$$= \int_{y=\infty}^{y=-\infty} \delta(-b(x-x_0))\, f(x)\,dx \; ; \quad \begin{matrix} y = -bx \\ dy = -b\,dx \end{matrix}$$

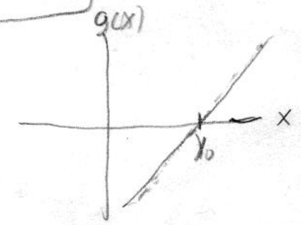
$$= -\frac{1}{b}\int_{\infty}^{-\infty} \delta(y+bx_0)\, f\left(\tfrac{y}{-b}\right) dy$$

$$= \frac{1}{c}\int_{\infty}^{-\infty} \delta(y-cx_0)\, f\left(\tfrac{y}{c}\right) dy$$

207f

$$= -\frac{1}{c}\int_{-\infty}^{\infty} \delta(y-cx_0)\, f\left(\tfrac{y}{c}\right) dy = -\frac{1}{c} f(x_0)$$

209a



209b

$$\int \delta(g(x))\, f(x)\, dx = \int \delta\left(\frac{dg}{dx}(x-x_0)\right) f(x)\, dx$$
$$= \frac{1}{\left|\frac{dg}{dx}(x_0)\right|}\, f(x_0)$$

209c

see 209b.

210a

$$\int \delta(\vec{r}-\vec{r}_0) f(\vec{r})\, dV = \int \delta(x-x_0)\delta(y-y_0)\delta(z-z_0) f(x,y,z)\, dx\, dy\, dz$$
$$= \left[\int \delta(y-y_0)\delta(z-z_0)\, dy\, dz\right]\left[\int \delta(x-x_0) f(x,y,z)\, dx\right]$$
$$= \int \delta(y-y_0)\delta(z-z_0) f(x_0,y,z)\, dy\, dz$$

210b

see 210a