The Case of the Identically Accelerated Twins

It is important to recognize that it is the relativity of simultaneity that is directly responsible for the rate at which the traveling twin ages. S. P. Boughn[21] has described an excellent variation of the twin problem that helps dispel the notion that the rate at which the traveler ages is a direct result of the acceleration he experiences and emphasizes the role of the synchronization of moving clocks in understanding many predictions of special relativity. We will describe a situation in which the twins receive identical accelerations yet age differently.

Suppose two twins, Dick and Jane, plan a journey that involves accelerating from their home inertial frame S into a new inertial frame S' moving at speed v. They have identical spaceships, each containing the same amount of fuel and parked on the x axis of S separated by a distance x_0 with Jane's to the right of Dick's, as shown in Figure 1-36*a*. They synchronize their clocks with those of their mom and dad, who will remain at home in S. At the appointed time on their 21st birthday they wave goodbye, start their engines, and accelerate off to the right along the x axis. After using all their fuel in equal time intervals, both spaceships have reached speed v (since they were identical and carried the same amount of fuel) and are coasting in S'. Comparing their ships' logs, Dick and Jane find that their accelerations were identical, but to their astonishment they discover that Jane is now older than Dick! In addition, their spaceships are now farther apart than when they started (Figure 1-36*b*). How can that be?

To understand the twins' unexpected observations, consider two events in S': the arrival of each of the twins. Suppose these occur on the birthdays of the twins (recall that they accelerated for identical time intervals). The Lorentz transformation gives us the time of these events in S', the twin's reference frame, to be

$$t'_D = \gamma(t_D - vx_D/c^2)$$
$$t'_J = \gamma(t_J - vx_J/c^2)$$

where v = speed of S' relative to S and the subscripts indicate the birthdays of Dick and Jane. The twins now measure a time difference between their birthdays (!) given by

$$t'_D - t'_J = \gamma(t_D - t_J) - \gamma v(x_D - x_J)$$

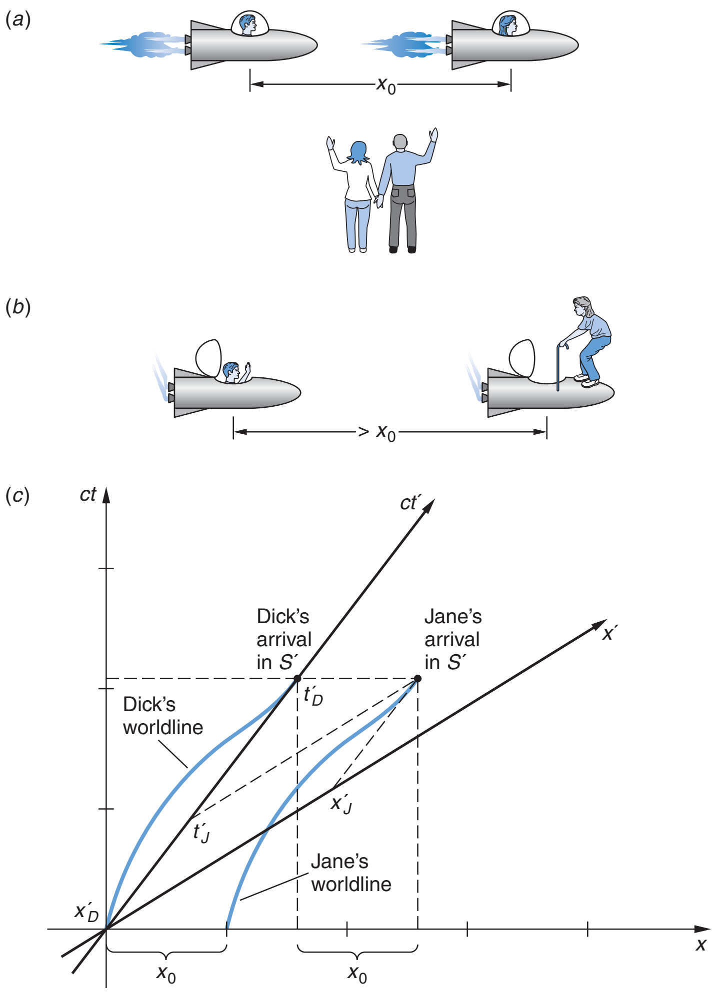

FIGURE 1-36 (*a*) The twins as they begin their journey. (*b*) The perplexed twins after arriving in S'. (*c*) The worldlines of Dick and Jane drawn in the inertial frame S show Jane's birthday/arrival in S', t'_J, to have occurred prior to Dick's, making her the older. For this diagram = 0.75.

The unprimed measurements made by Mom and Dad in S are, of course, $t_D - t_J = 0$ and $x_J - x_D = x_0$, or

$$t'_D - t'_J = \gamma v x_0/c^2$$

Thus, according to the twins' measurements, Jane's birthday occurred at t'_J, a time $\gamma v x_0/c^2$ *before* Dick's. In addition, they now find their separation to be

$$x'_J - x'_D = \gamma(x_J - x_D) - \gamma v(t_J - t_D)$$

$$\text{or} \quad x'_J - x'_D = \gamma x_0$$

Figure 1-36*c* illustrates the twins' worldlines and shows clearly the results just computed.

That the twins age differently even though their accelerations were identical seems paradoxical. But, in fact, there is no paradox. As was the case with Homer and Ulysses, the twins' situations are not identical. Jane started x_0 to the right of Dick. Therefore, even though their clocks were synchronized in S, they were unsynchronized by an amount $\gamma v x_0/c^2$ for observers in S' moving at speed v with respect to S. When the twins reach S', they find that indeed, this is exactly the difference in their ages. Once again, the key to resolving the paradox is understanding the relativity of simultaneity.