Lecture #27 Summary
PHYS 313 Electricity and Magnetism I

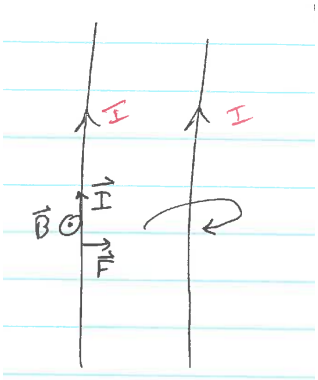
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We are now starting on the topic of Magnetostatics. This is the study of magnetic fields created by steady electrical currents. What we will explore today has nothing to do with Coulomb's law. Electrical currents involve only charge-neutral objects, and no degree of polarization.

We will see that there are entirely new forces created by electrical currents, and discover that a new field, the magnetic field, conveys these forces.

Force between Current-Carrying Wires
Demo K1-01



This demonstration shows that two parallel electrical currents create forces on each other. There is an attractive force for parallel currents and a repulsive force for anti-parallel currents.

1) There is no Coulomb electric force at play here.

2) One can propose that a magnetic field $\vec{B}$ is created by the current and given by a right hand rule.

3) The resulting force on the other current conveyed by the magnetic field is $\vec{F}_{mag} \sim \vec{I} \times \vec{B}$ created by the magnetic field $\vec{B}$ interacting with $\vec{I}$.



K1-01: FORCE BETWEEN CURRENT-CARRYING WIRES

PURPOSE: To demonstrate the force between two adjacent parallel current-carrying wires.
DESCRIPTION: A six-foot tall support holds two wires which can be connected for parallel or anti-parallel currents. The motion is clear: attractive for parallel wires and repulsive for anti-parallel wires.

Magnetic Fields

We would like to develop our understanding of magnetostatics in exactly the same way as electro-statics.

> Magnetostatics: The study of magnetic fields produced by steady (time-independent) electric currents.

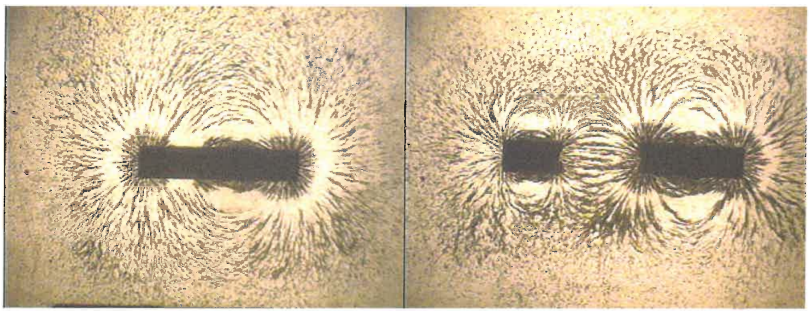
However, it is hard to formulate things in magneto-statics starting from the magnetic field produced by one moving charge, and then use superposition to understand a steady current.
A single moving charge is in no way a good approximation to a steady current.

In electrostatics we went this way
1) Electric Force due to 1 charge (Coulomb's Law)
2) Electric Field due to 1 charge (Source of $\vec{E}$)
3) Linear superposition to describe continuous charge distribution
4) $\vec{\nabla}\cdot\vec{E}=\rho/\epsilon_0$, $\vec{\nabla}\times\vec{E}=0$ (statics) + BCs uniquely determine $\vec{E}(\vec{r})$ (source of $\vec{E}$)
5) Electric Potential (since $\vec{\nabla}\times\vec{E}=0$ in statics) etc. $\vec{E}=-\vec{\nabla}V$
In magnetostatics we can not really follow this procedure. The key difference: $\vec{B}$ **has no scalar source**, only vector sources!

Our Procedure is this:
(1) Introduce the Magnetic force on a moving charge
(2) Introduce Electric Currents (a source of magnetic fields)
(4) Biot-Savart Law - How currents produce magnetic fields
(4) $\vec{\nabla}\cdot\vec{B}=0$ (no point sources of magnetic field), $\vec{\nabla}\times\vec{B}=\mu_0\vec{J}$ (source of $\vec{B}$) + BC's determine $\vec{B}(\vec{r})$.

$$\boxed{\vec{\nabla}\cdot\vec{B}=0} \qquad \boxed{\vec{\nabla}\times\vec{B}=\mu_0\vec{J}}$$

(5) Magnetic Vector Potential $\vec{A}$: $\boxed{\vec{B}=\vec{\nabla}\times\vec{A}}$

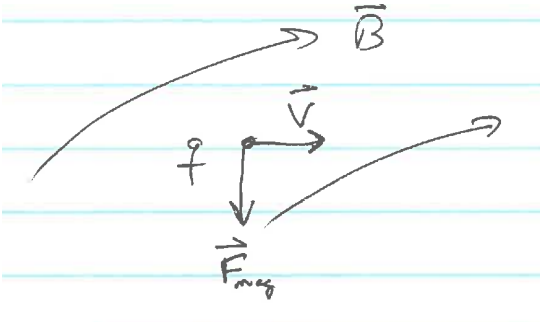


J5-06: MAGNET - BROKEN BUT NO MONOPOLE

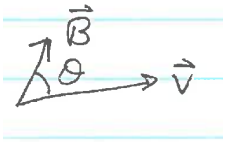
PURPOSE: Demonstrate that magnet poles come in pairs.
DESCRIPTION: The magnetic field of a bar magnet is demonstrated using iron filings on an overhead projector. The magnet is then broken and the demonstration is repeated. Each half of the original magnet has both poles, as can be easily seen.

Objects with electric polarization $\vec{P}$ will do the same thing. The difference is that you will eventually get to electric monopoles if you can continue to break an electrically polarized object into smaller pieces.

What is the Magnetic Force exerted on a Moving Charge?
One finds from experiments that a moving charge in a magnetic field experiences a force with the characteristics:



(1) $|\vec{F}_{mag}| \propto q$ and the force on a negative charge is in the opposite direction to the force on a positive charge
(2) $|\vec{F}| \propto |\vec{v}|$ where $\vec{v}$ is the velocity of the charge. A charge at rest experiences no magnetic force
(3) $\vec{F} \perp \vec{v}$ and $\vec{B}$
(4) $|\vec{F}| \propto \sin\theta$ where

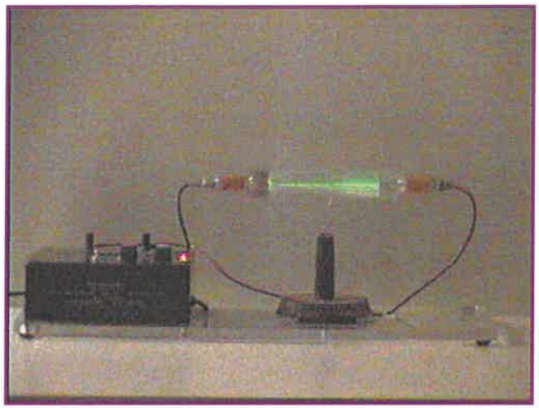


θ is the angle between $\vec{B}$ and $\vec{v}$.
All of these observations can be summarized as follows:

$$\boxed{\vec{F} = q\,\vec{v} \times \vec{B}}$$

$$|\vec{F}| = q\,|\vec{v}|\,|\vec{B}|\,\sin\theta$$

All points verified.



K1-12: CATHODE RAY TUBE - DEFLECTION BY MAGNET

PURPOSE: To demonstrate visually the force on an electron beam by a magnetic field.
DESCRIPTION: A cathode ray tube produces a beam of electrons which can be seen as it hits a phosphorescent surface inside the tube. Holding a magnet close to the CRT parallel to the table so that it produces a horizontal magnetic field inside the tube causes the electron beam to deflect up or down. The vector nature of the force can be studied by reversing the magnet.

If we throw in an electric field, we get

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$ Lorentz Force Law.

Electric Currents

Electric Currents are one source of magnetic fields. This is the only source we consider in Magnetostatics.

Current is the rate at which charge passes a given point in space:

$$I \equiv \frac{\Delta Q}{\Delta t}$$

It has units of Amperes = 1 Coulomb/second (C/s).

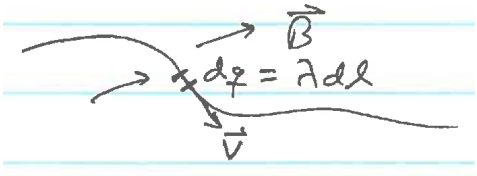
One can also think of a current in a filamentary wire as a charge per unit length λ moving at some velocity $\vec{v}$

$$\boxed{\vec{I} = \lambda \vec{v}}$$



Suppose a current flows in the presence of a magnetic field? Then there is a force exerted on each charge making up the current

$$\vec{F} = q\,\vec{v} \times \vec{B}$$



$$\longrightarrow \int_{\text{current path}} dq\,\vec{v} \times \vec{B}$$

But $dq = \lambda\, dl$

$$\vec{F} = \int_{\text{line}} \lambda\,\vec{v} \times \vec{B}\, dl$$

Using the other definition of current, gives

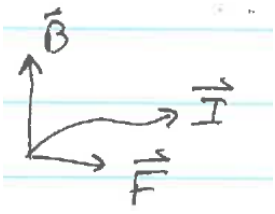
$$\vec{F} = \int_{\text{current path}} (\vec{I} \times \vec{B})\, d\ell$$

But since $\vec{I}$ and $d\vec{\ell}$ both point in the same direction, we can say

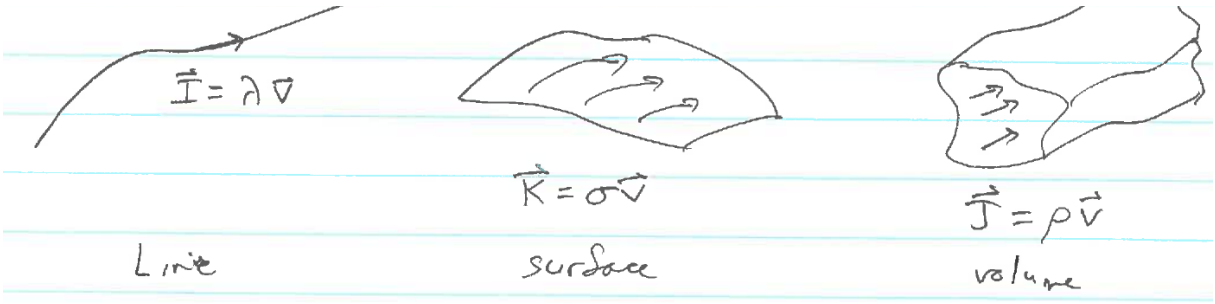
$$\vec{F} = \int_{\text{current path}} I\,(d\vec{\ell} \times \vec{B})$$

I could be a function of position along the wire, but usually it is not. Hence we can write

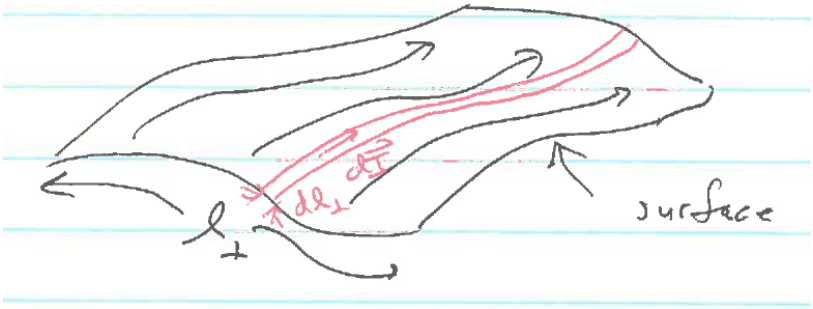
$$\boxed{\vec{F} = I \int_{\text{current path}} d\vec{\ell} \times \vec{B}}$$



One can also have currents that flow on surfaces and in volumes (just as one can have charge on surfaces, σ and in volumes ρ).



The surface current density is the current per unit width perpendicular to the flow



$$\vec{K} \equiv \frac{d\vec{I}}{d\ell_\perp} \simeq \frac{\vec{I}}{\ell_\perp} \qquad \text{for a finite width}$$

It can be thought of as the mobile surface charge density having a velocity $\vec{v}$ along the surface

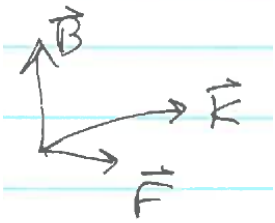
$$\vec{K} = \sigma \vec{v}$$

The magnetic force on a surface current is

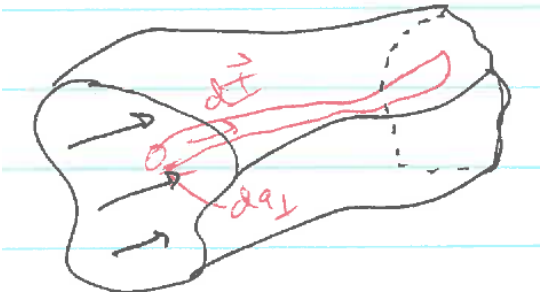
$$\vec{F} = \int_{\text{surface}} (\vec{v} \times \vec{B})\, dq \qquad dq = \sigma\, da$$

$$= \int_{\text{surface}} \sigma\, \vec{v} \times \vec{B}\, da$$

$$\boxed{\vec{F} = \int_{\text{surface}} \vec{K} \times \vec{B}\, da}$$



The volume current is the current per unit area perpendicular to the flow.



$$\vec{J} \equiv \frac{d\vec{I}}{da_\perp} \simeq \frac{\vec{I}}{a_\perp} \qquad \text{for a uniform current flow.}$$

This can be thought of as a charge density moving along at some velocity $\vec{v}$:

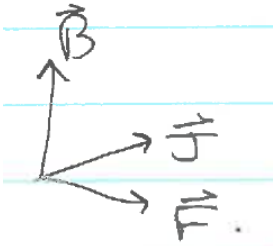
$$\vec{J} = \rho\,\vec{v}$$

The magnetic force on a volume current is

$$\vec{F} = \int_{\text{volume}} \vec{v} \times \vec{B}\, dq \qquad dq = \rho\, d\tau$$

$$= \int_{\text{volume}} \rho\,\vec{v} \times \vec{B}\, d\tau$$

$$\boxed{\vec{F} = \int_{\substack{\text{volume} \\ \text{of the flow}}} \vec{J} \times \vec{B}\, d\tau}$$

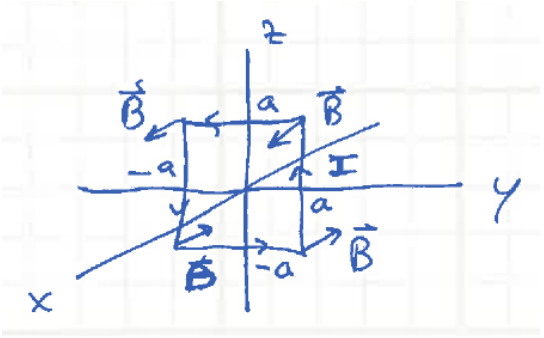


Problem 5.4

The magnetic field in some region has the form

$$\vec{B} = hz\,\hat{x}$$

Find the force on a square loop (side a), lying in the yz-plane and centered at the origin, if it carries a current I, flowing counter clockwise, when you look down the x-axis.



The current is in 4 segments

① $z = a/2$ $\quad y = +\dfrac{a}{2}$ to $-a/2$ and $d\ell = -dy$

$$\vec{F} = \int_{\text{line 1}} \vec{I} \times \vec{B}\, d\ell = \int_{+a/2}^{-a/2} (-\hat{y}\, I) \times (ha/2)\, \hat{x}\, (-dy) = +\frac{Iha}{2} \int_{+a/2}^{-a/2} dy\, \hat{y} \times \hat{x}$$

$$= -\frac{Iha}{2}\, \hat{z}\, y\Big|_{+a/2}^{-a/2} = +\frac{Iha^2}{2}\, \hat{z} \quad \text{This force is upward, in the } +z \text{ direction.}$$

② $y = -a/2$ $\quad z = a/2 \to -a/2$, and $d\ell = -dz$

$$\vec{F} = \int_2 \vec{I} \times \vec{B}\, d\ell = \int_{a/2}^{-a/2} (-I\hat{z}) \times (hz\, \hat{x})\, (-dz) = -Ih \int_{-a/2}^{a/2} \hat{z} \times \hat{x}\, z\, dz$$

$$= -Ih\, \hat{y} \int_{-a/2}^{a/2} z\, dz = -Ih\, \hat{y}\, \frac{z^2}{2}\Big|_{-a/2}^{a/2} = 0$$

③ $z = -a/2$ $\quad y = -a/2$ to $a/2$, and $d\ell = +dy$

$$\vec{F} = \int_3 \vec{I} \times \vec{B}\, d\ell = \int_{-a/2}^{a/2} (I\, \hat{y}) \times \left(-\frac{ha}{2}\, \hat{x}\right) (+dy) = -\frac{Iha}{2}\, \hat{y} \times \hat{x} \int_{-a/2}^{a/2} dy$$

$$= \frac{Iha}{2}\, \hat{z}\, y\Big|_{-a/2}^{a/2} = \frac{Iha^2}{2}\, \hat{z} \quad \text{This force is also upwards, like that on segment ①.}$$



④ $y = a/2$ $\quad z = -a/2 \to a/2$, and $d\ell = +dz$

$$\vec{F} = \int_4 \vec{I} \times \vec{B}\, d\ell = \int_{-a/2}^{a/2} (I\hat{z}) \times (hz\, \hat{x})\, (+dz) = Ih\, \hat{z} \times \hat{x} \int_{-a/2}^{a/2} z\, dz$$

$$= Ih\, \hat{y}\, \frac{1}{2} z^2\Big|_{-a/2}^{a/2} = 0$$

The total force is

$$\vec{F} = \frac{Iha^2}{2}\, \hat{z} + \frac{Iha^2}{2}\, \hat{z}$$

$$\boxed{\vec{F} = Iha^2\, \hat{z}} \qquad \text{upward in the } +z\text{-direction}$$