Lecture #1 Summary
PHYS 313 Electricity and Magnetism I

Prof. Steven Anlage

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Slides and video recoding of lecture

The slides shown during this lecture, along with a recording of the lecture, are available on the open class website here: Phys313 under Lecture 1.

Electromagnetism

We shall study classical electromagnetism in the non-relativistic limit.

The separate fields of electricity and magnetism were unified by Maxwell and Lorentz in the late 19^{th} century.

Electromagnetism is a Field Theory

The space around electric charges is filled with electric and magnetic fields created by these charges. It is an electromagnetic "odor".

Other charges then experience forces. These forces are conveyed by the electric and magnetic fields.

The forces that charges and currents exert are what we can measure. We use test charges to convert fields into the measurable quantity: force. We substitute fields to stand in for the forces that are measurable.

Coulomb's Law and the Electric Field

Charge q_1 exerts a force on charge q_2:

$$\vec{F}_{12} = \frac{q_1 q_2\, k_e}{|\vec{r}_{12}|^2}\,\hat{r}_{12} \tag{1}$$

We can think of this force as being due to an electric field $\vec{E}$ created by charge q_1:

$$\vec{E}(\vec{r}) = \frac{k_e q_1}{|\vec{r}|^2}\,\hat{r} \tag{2}$$

where $\vec{r}$ is the vector from q_1 to some arbitrary point in space.

A test charge q_2 will experience a force:

$$\vec{F}_{12} = q_2\vec{E} \tag{3}$$

However, this test charge should be small enough so that the Newton's Third Law reaction force on q_1 isn't so large as to disturb it. This illustrates the limitation of the field concept.

Field Theory Approach

Instead of looking at the forces between charges and currents, we will instead focus on the theory of the fields themselves. The forces can always be recovered from the fields.

To create a theory of fields, we will rely heavily on differential and integral calculus. Your Math and Physics courses should have prepared you for this work. However, we are going to review the highlights of the math you are expected to know.

Review of Mathematics

Vectors

A vector is a quantity with both magnitude and direction. Vector $\vec{A}$ has magnitude $|\vec{A}|$.

Scalar Product

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta \tag{4}$$

where θ is the angle between the vectors when brought tail to tail.

It is a "projection" or measure of co-incidence of the two vectors. This concept generalizes to quantum states in infinite-dimensional Hilbert space.

Cross Product

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta\, \hat{n} \tag{5}$$

where $\hat{n}$ is the unit vector perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$.

In component form, the cross product is given by a determinant (Cartesian Coordinates):

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{6}$$

Remember how to expand 3x3 determinants!

More Complicated Products

$\vec{A} \cdot (\vec{B} \times \vec{C})$ — volume of the parallelepiped

BAC-CAB Rule:

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \tag{7}$$

Griffiths' Separation Vector

Griffiths uses a script $\vec{\mathscr{r}}$ to denote the separation between a source of field and the point where the field is evaluated:

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' \tag{8}$$

where $\vec{r}\,'$ is the source point (charge, current, etc.) and $\vec{r}$ is the field point (where the field is evaluated). This notation is a choice that is utilized throughout the textbook.

Differential Calculus

Derivative

The derivative of a function of one variable, $f(x)$, is the slope at a particular point: $\frac{df}{dx}$

Gradient

The gradient of a scalar function of two or more variables is a vector quantity.

- Its direction is the direction of maximum increase of the function.
- Its magnitude is the slope (rate of increase) of the function in that direction.

Example: $h(x, y)$ may be the altitude of a point at location (x, y) in two-dimensions. $\vec{\nabla} h$ points in the direction of maximum increase in altitude, while $|\vec{\nabla} h|$ gives the rate of change of altitude in that direction.

The Gradient Operator

$$\vec{\nabla} Q = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \right) Q(x,y,z) \tag{9}$$

$\vec{\nabla}$ is an operator. Operators interrogate the functions, vectors, etc. that follow them on the right. In this case, the grad operator bites a scalar function (Q in this case) and creates a vector function of (x, y, z).

Three Ways the Gradient Operator Can Act

1. Bite a scalar function: $\vec{\nabla} Q(x,y,z)$ — this is called the **gradient**
2. Dot a vector function: $\vec{\nabla} \cdot \vec{V}(x,y,z)$ — this is called the **divergence**
3. Cross product of a vector function: $\vec{\nabla} \times \vec{V}(x,y,z)$ — this is called the **curl**

The Divergence

$$\vec{\nabla} \cdot \vec{V} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \right) \cdot (V_x\hat{x} + V_y\hat{y} + V_z\hat{z}) \tag{10}$$

$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \tag{11}$$

since $\hat{x} \cdot \hat{y} = \hat{x} \cdot \hat{z} = \hat{y} \cdot \hat{z} = 0$ and $\frac{\partial}{\partial x}\hat{y} = \frac{\partial}{\partial x}\hat{z} = \cdots = 0$. Cartesian coordinates are unique in this regard. The so-called curvilinear coordinate systems (cylindrical and spherical) have unit vectors that vary with location in space, complicating things a bit.

The divergence $\vec{\nabla} \cdot \vec{V}$ is a measure of how much a vector function $\vec{V}$ spreads out (diverges) from a given point in question.

The Curl

The curl of a vector field $\vec{v}(x,y,z)$ in Cartesian coordinates is given by the determinant:

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix} = \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) - \hat{y}\left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} \right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right)$$

The curl $\vec{\nabla} \times \vec{v}$ of a vector function $\vec{v}$ is a measure of how much the vector $\vec{v}$ curls around a given point.

One way to visualize this is to put a cork with paddles into a body of water flowing with velocity field $\vec{v}$. If the paddlewheel rotates, then there is curl at that point.

A whirlpool or vortex has large curl. See the slides for lecture 1 for examples.