

Lecture #19 Summary

PHYS 313 Electricity and Magnetism I

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Dielectrics and Electric Fields in Matter

We said that a conductor is a perfect dielectric. It excludes all of the electric field from its interior in Electrostatic Equilibrium (EE).

Ordinary (Non-Perfect) Dielectrics only do a partial job of excluding electric field from their interior. Note that we treat these dielectric materials as insulators, so that they have no free charge, but only *bound* charge.

A homogeneous, uniform dielectric material will reduce an external electric field $\vec{E}_{ext}$ to an internal value given by

$$\boxed{\vec{E}_{int} = \frac{\vec{E}_{ext}}{\epsilon_r}} \quad \vec{E}_{int} = 0 \text{ for a perfect dielectric}$$

where ϵ_r is the relative dielectric constant of the material, a positive finite number to be defined later.

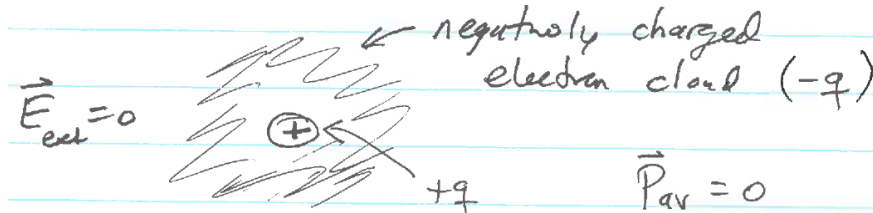
Now we are talking about insulators. These are materials with no free charge that can move about to screen out an electric field. The charges are all bound to atoms. Hence they have difficulty in reacting to an applied field.

The bound charges can do two things in response to an external field

- (1) Stretch - i.e. separate positive and negative charge
- (2) Rotate - To minimize the interaction energy

(1) Let's first look at stretching at the atomic scale.

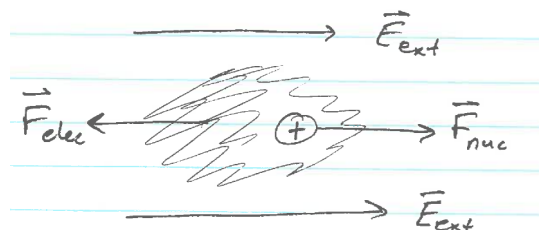
Consider a neutral atom with a positively charged nucleus and a negatively charged electron cloud. With no electric field applied, it has no net electric dipole moment, on average.



$$\begin{aligned} \vec{p} &= q \int \psi^* \vec{r} \psi dV = 0 \quad \text{for all the hydrogen atom wavefunctions } \psi. \\ &= q (\langle x \rangle \hat{x} + \langle y \rangle \hat{y} + \langle z \rangle \hat{z}) \end{aligned}$$

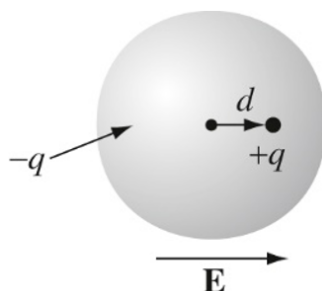
(This $\vec{p}$ is not the quantum mechanical momentum operator!)

Now if an external electric field is applied the nucleus and the electron cloud experience equal and opposite net forces. This separates the two



This is called the Stark Effect in quantum mechanics.

The external electric field has produced an electric dipole moment in the atom



where q is the nuclear ($-$ total electronic) charge, and $\vec{d}$ is the displacement vector created when the external force is balanced again by the electron-nucleus attractive force.

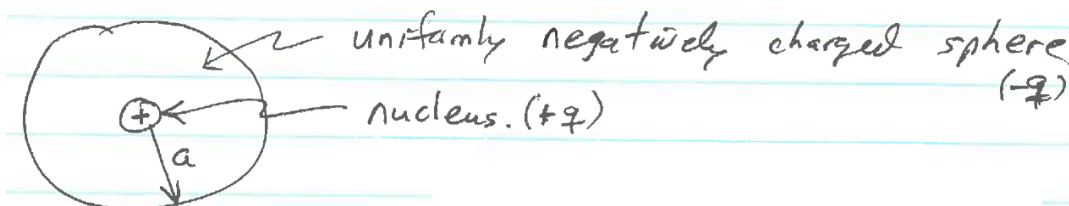
Note that $\vec{p}$, the electric dipole moment, points in the same direction as $\vec{E}$. Although not always true, the dipole moment is often found to be proportional to the electric field;

$$\vec{p} = \alpha \vec{E}$$

Empirical Fact for many dielectrics

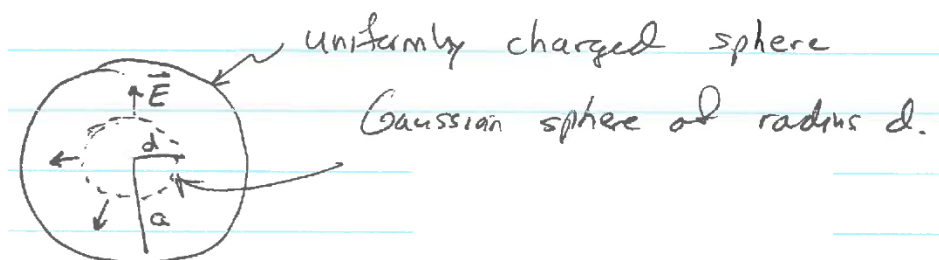
where α is called the “polarizability” of the dielectric object/material.

We can estimate the polarizability of an atom using a simple spherical model. Assume the electrons constitute a uniformly charged sphere of radius a



The nucleus feels $\vec{E} = 0$ due to being at the center of the spherical electron cloud.

In the presence of an external field, there is a displacement of the nucleus relative to the center of the sphere. At that point it experiences a net electric field from the electrons, which we can find from Gauss's Law.



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$E(d) 4\pi d^2 = \frac{1}{\epsilon_0} (-q) \frac{\frac{4}{3}\pi d^3}{\frac{4}{3}\pi a^3} = \frac{1}{\epsilon_0} (-q) \frac{d^3}{a^3}$$

Hence

$$E(d) = \frac{-q d}{4\pi\epsilon_0 a^3}$$

It's a linear restoring force! $F = kd$ $\omega = \sqrt{k/m}$

The force exerted by this field on the nucleus must balance that due to the external field.

$$qE(d) = qE_{ext}$$

We can use the expression for the polarizability to find α for an atom

$$\frac{qd}{4\pi\epsilon_0 a^3} = E_{ext}$$

or

$$p = qd = \alpha E_{ext} = (4\pi\epsilon_0 a^3) E_{ext}$$

So

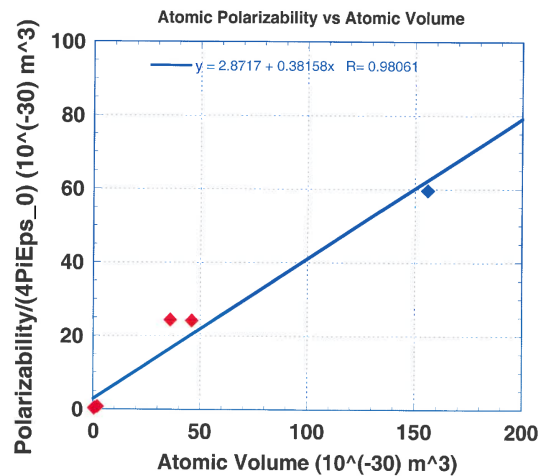
$$\alpha = 4\pi\epsilon_0 a^3 = \left(\frac{4}{3}\pi a^3\right) 3\epsilon_0 = 3\epsilon_0 V_{atom}$$

The polarizability is proportional to the volume of the atom! Here is a Table of atoms and their atomic polarizability.

Atom	Volume* 10^{-30} m^3	Polarizability $\alpha/4\pi\epsilon_0$ 10^{-30} m^3
H	2.1	0.667
He	0.5	0.205
Li	36	24.3
Ne	0.6	0.396
Na	46	24.1
Cs	156	59.6

* $V = \frac{4}{3}\pi r^3$ r = atomic radius (Å) from the periodic table.

See the plots below of α vs. atomic volume, and the log of atomic polarizability vs. nuclear charge, below.



How big is the distortion of the atom, d , in a modest field of $E = 10^6 \text{ V/m}$?

$$p = qd = \alpha E_{ext}$$

$$d = \frac{\alpha}{q} E_{ext} \sim \frac{(1)(4\pi) \left(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}\right) 10^{-30} \text{ m}^3 \cdot 10^6 \frac{\text{V}}{\text{m}}}{1.602 \times 10^{-19} \text{ C}}$$

$$= 7 \times 10^{-16} \text{ m} \quad \text{Units: } C = \frac{Q}{V} \quad \frac{\text{m}^3 \text{ F V}}{\text{m}^2 \text{ C}} = \text{m}$$

Hence the displacement of the charges d is on the order of a nuclear diameter!

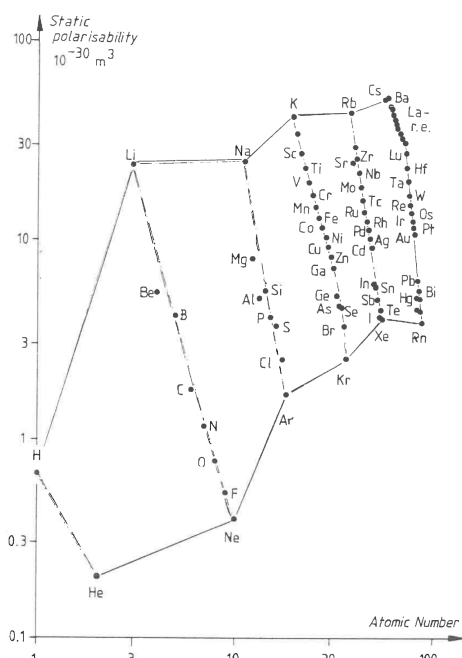
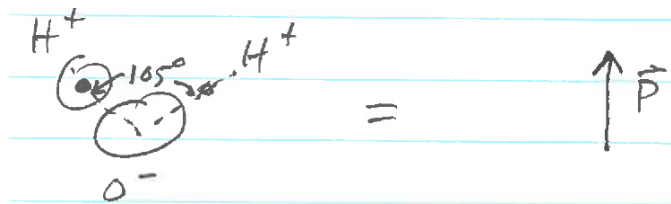


Figure 2.5 The logarithm of the static atomic polarisabilities plotted against the logarithm of the atomic number, showing the periodicity and regularity of the relationship. Note how the polarizability shrinks with atomic volume going from left to right in each row of the periodic table. Data from Teachout and Pack (1971).

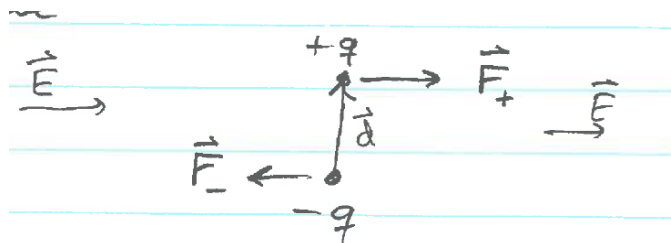
Forces on Polar Molecules

Many molecules possess an intrinsic electric dipole moment $\vec{p}$ due to charge distribution inside the molecule. H_2O is a famous example



Although the molecule is charge neutral (zero monopole moment), it has a large electric dipole moment. This makes it a universal solvent.

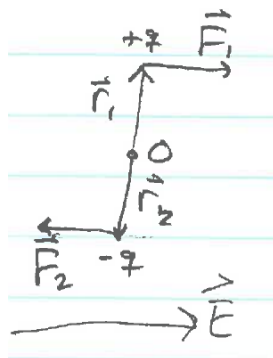
Under an external electric field, the molecule feels no net force, but it does experience a net torque:



$$\vec{F}_+ + \vec{F}_- = 0$$

The torque about the center of the molecule is:

$$\vec{\tau}_{net} = \sum_{i=1}^N \vec{r}_i \times \vec{F}_i$$



$$\begin{aligned}\vec{\tau}_{net} &= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 \\ &= \frac{\vec{d}}{2} \times (q\vec{E}) + \left(\frac{-\vec{d}}{2}\right) \times (-q\vec{E}) \\ \vec{\tau}_{net} &= q\vec{d} \times \vec{E}\end{aligned}$$

This property of H₂O makes it very susceptible to being polarized by an electric field. This is exploited by the microwave oven to convert electromagnetic energy into thermal energy (i.e. to heat your food from the inside).

But since $\vec{p} = q\vec{d}$, we can write

$$\boxed{\vec{\tau}_{net} = \vec{p} \times \vec{E}}$$

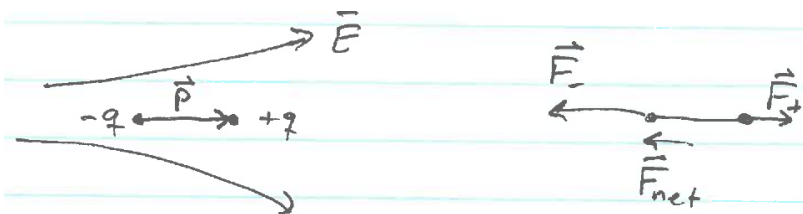
The torque tends to align the dipole moment parallel to the external electric field.

If the molecule is free to rotate it will. Otherwise it will be stuck and just wiggle back and forth as the electric field changes sign.

A net force can be exerted on a dipole if the electric field is inhomogeneous in space, such that the positive and negative parts of the dipole experience different forces. Clearly this requires some divergence of the electric field in the direction of the dipole moment $\vec{p}$. One finds that the net force is given by,

$$\boxed{\vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}} = p_x \frac{\partial \vec{E}}{\partial x} + p_y \frac{\partial \vec{E}}{\partial y} + p_z \frac{\partial \vec{E}}{\partial z}$$

in Cartesian coordinates.



So far we have only discussed individual atoms and molecules. What happens when we have lots of them? They all do the same thing and their individual dipole moments line up and produce a bigger dipole moment. The material as a whole becomes “polarized” by the applied electric field.

Define the polarization ($\vec{P}$) as the dipole moment per unit volume of the material.

$$\boxed{\vec{P} = \text{polarization} = \frac{\text{dipole moment}}{\text{volume}}}$$

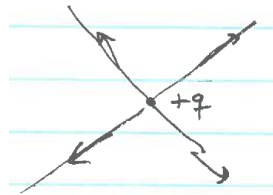
Units

$$|\vec{P}| \sim \frac{qd}{V} = \frac{\text{Coulomb}}{\text{m}^2} \sim \sigma$$

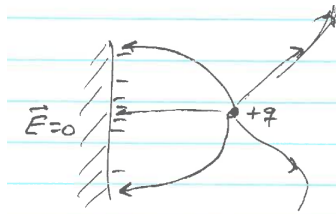
Polarization has units of charge per unit area! This idea will return when we talk about bound charge at the surface of a polarized dielectric.

Once an object becomes polarized, what kind of electric field does it produce inside and outside itself?

In the case of a perfect dielectric (conductor in EE), we found that $\vec{E} = 0$, $V = \text{constant}$, inside a bulk conductor and that the field outside was strongly influenced by the induced charges on the surface.



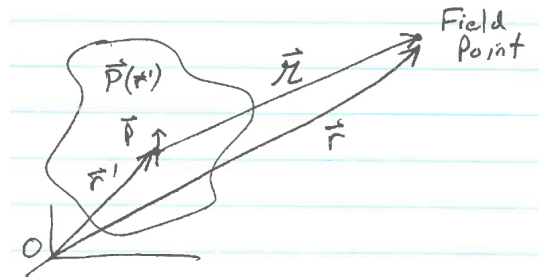
charge in free space



charge next to a grounded conductor
 $\Rightarrow$ Method of images

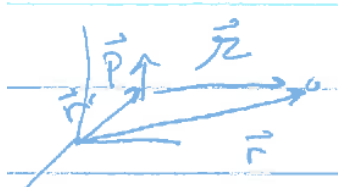
Now let's study the case of a regular dielectric material with finite ϵ_r to see what happens to the electric field inside and outside a polarized object.

Given a polarized material with known polarization $\vec{P}$, what fields does it produce around and inside itself?



The object is assumed to be charge neutral, but polarized.

Chop the material into a whole bunch of elementary dipoles $\vec{p}$. Each produces a potential given by,



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$$

The potential of an elementary dipole falls like $1/r^2$, and the field falls like $1/r^3$
 To construct the total potential due to a macroscopic polarized object, integrate

$$\vec{p} \Rightarrow \vec{P} d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\hat{z} \cdot \vec{P}(\vec{r}') d\tau'}{z^2}$$

Note that this is different from the $\vec{E}$ and V calculations that we have done before on monopole charge!

This integral can be transformed into something more useful as follows

Note that

$$\frac{\hat{z}}{z^2} = \vec{\nabla}' \left(\frac{1}{z} \right)$$

$\vec{\nabla}'$ means gradient w.r.t. the source point coordinates $\vec{r}'$

Check: What is $\vec{\nabla}' \left(\frac{1}{z} \right)$?

Do it in Cartesian

$$z = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} = |\vec{r} - \vec{r}'|$$

$$\begin{aligned} \vec{\nabla}' \left(\frac{1}{z} \right) &= \frac{\partial}{\partial x'} [(x-x')^2 + (y-y')^2 + (z-z')^2]^{-1/2} \hat{x}' \\ &\quad + \frac{\partial}{\partial y'} [\]^{-1/2} \hat{y}' + \frac{\partial}{\partial z'} [\]^{-1/2} \hat{z}' \\ &= -\frac{1}{2} 2(x-x') [\]^{-3/2} \hat{x}' + (y-y') [\]^{-3/2} \hat{y}' + (z-z') [\]^{-3/2} \hat{z}' \\ &= + \frac{\vec{z}}{z^3} = \frac{\hat{z}}{z^2} \end{aligned}$$

$$\vec{\nabla}' \left(\frac{1}{z} \right) = \frac{\hat{z}}{z^2}$$

Also, note that if the gradient is with respect to the *field* point, there is a minus sign:

$$\vec{\nabla} \left(\frac{1}{z} \right) = -\frac{\hat{z}}{z^2}$$

Back to the potential calculation for the polarized material:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \vec{P}(\vec{r}') \cdot \vec{\nabla}' \left(\frac{1}{z} \right) d\tau'$$

Now use product rule #5 from the inside front cover of the textbook:

$$\vec{A} \cdot (\vec{\nabla} f) = \vec{\nabla} \cdot (f \vec{A}) - f(\vec{\nabla} \cdot \vec{A}) \quad \text{where } \vec{A} = \vec{P} \quad \text{and } f = \frac{1}{z}$$

as

$$\vec{P} \cdot \vec{\nabla}' \left(\frac{1}{z} \right) = \vec{\nabla}' \cdot \left(\frac{\vec{P}}{z} \right) - \frac{1}{z} (\vec{\nabla}' \cdot \vec{P})$$

to get

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left[\int_{\mathcal{V}} \vec{\nabla}' \cdot \left(\frac{\vec{P}}{z} \right) d\tau' - \int_{\mathcal{V}} \frac{1}{z} \vec{\nabla}' \cdot \vec{P} d\tau' \right]$$

Use the divergence theorem on the first term:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left[\oint_S \frac{1}{z} \vec{P} \cdot d\vec{a}' - \int_{\mathcal{V}} \frac{1}{z} \vec{\nabla}' \cdot \vec{P} d\tau' \right]$$

and S is the surface of the polarized object which occupies the volume $\mathcal{V}$.

Now note that these look like potentials due to charges because they fall off like $1/z$ rather than the $1/z^2$ we started with!

The potential due to a polarized object is that due to a “new” kind of charge - “bound charge”

Look at the two integrands carefully

(1) Surface Integral $\frac{1}{z} \vec{P} \cdot d\vec{a}' = \frac{1}{z} \vec{P} \cdot \hat{n} da'$

Define a bound surface charge density as

$$\sigma_b = \vec{P} \cdot \hat{n}$$

$\hat{n}$ = outward normal vector to surface. See an example of surface bound charge in the Lecture 19 slides.

This is the bound surface charge, or charge per unit area arising at the surface of the polarized object.

The first term is the potential produced by this surface charge density σ_b .

(2) Volume Integral
$$\frac{1}{\epsilon_0} \vec{\nabla}' \cdot \vec{P}(\vec{r}') d\tau'$$

Define

$$\boxed{\rho_b \equiv -\vec{\nabla} \cdot \vec{P}}$$

as the *bound* charge per unit volume inside a polarized object. See an example of volume bound charge in the Lecture 19 slides.

The second term is the potential due to the bound charge inside the object (we are using the fact that $\vec{\nabla} = -\vec{\nabla}'$).

We can re-write the potential as

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma_b(\vec{r}')}{\epsilon_0} da' + \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\rho_b(\vec{r}')}{\epsilon_0} d\tau'}$$

where $\mathcal{V}$ is the volume of the polarized object and S is its surface. This reduces the problem to one that we have solved before - the potential due to surface and volume charges.