

Lecture #13 Supplement on Laplace's Equation

PHYS 313 Electricity and Magnetism I

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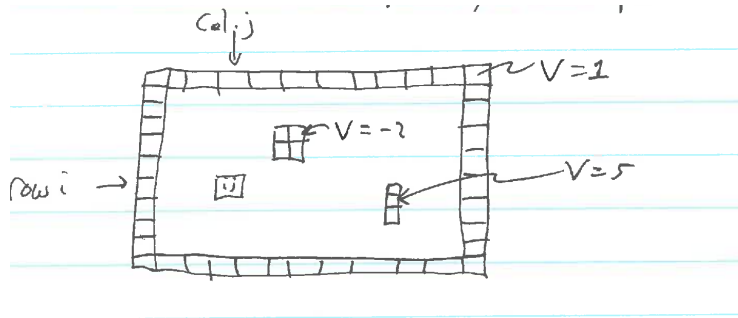
Fall 2026

1 Solving Laplace's equation using spreadsheets

- Open a spreadsheet and start on Sheet1.
- Create the boundaries and specify the potential there.

$V_{i,j} = 100$ for example {type = 100 in the appropriate cells}

One can also place conductors at fixed potential inside the region of interest.



- Calculate the potential at all interior points using the averaging property of solutions to Laplace's equation:

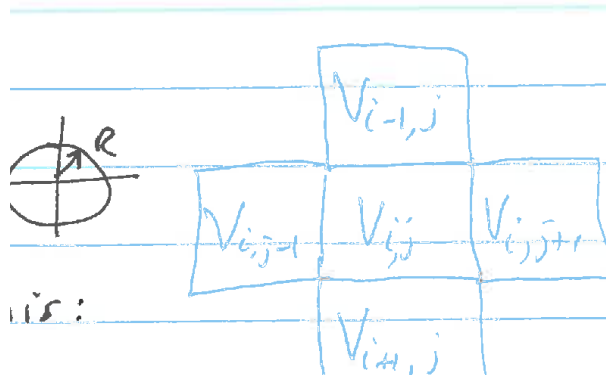
$$\text{three dimensions: } V(\vec{r}) = \frac{1}{4\pi R^2} \oint_{\text{sphere}} V dA$$

$$\text{two dimensions: } V(\vec{r}) = \frac{1}{2\pi R} \oint_{\text{circle}} V d\ell$$

In Excel, this can be done several different ways, such as:

$$V_{i,j} = (V_{i,j-1} + V_{i,j+1} + V_{i-1,j} + V_{i+1,j}) / 4$$

{e.g. at the cell C3, type =(C4+C2+B3+D3)/4. Copy this cell and paste it everywhere inside the spreadsheet that does not already have a potential specified}



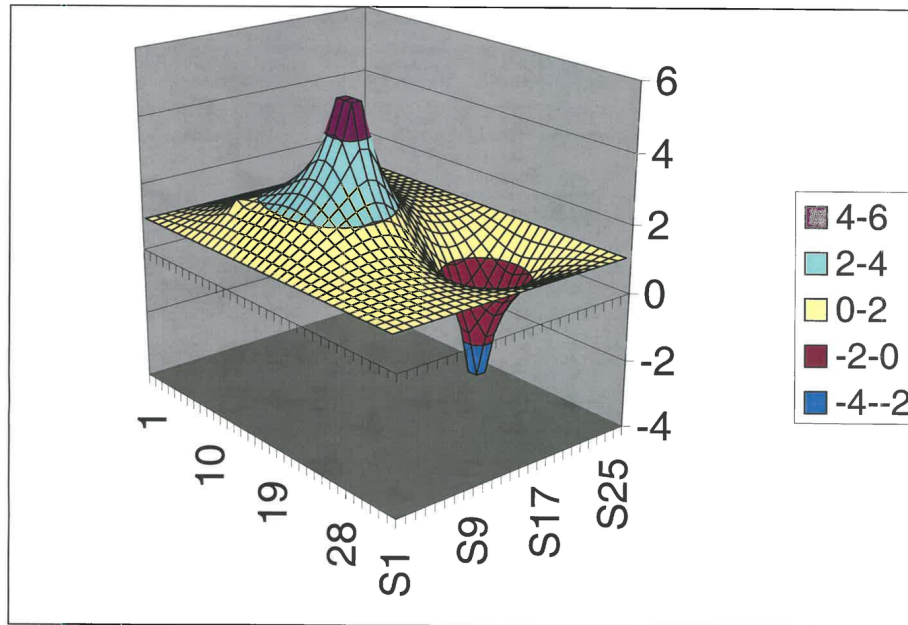
- d) Iterate the Sheet. The spreadsheet will iteratively solve Laplace's equation by the "relaxation method" and converge to the unique solution.

{Go to the "File" menu, click on "Options", then click on the "Formulas" menu. Check the box marked "Enable iterative calculation". You can control the number of iterations performed, or the tolerance for the maximum change in the spreadsheet before the calculation stops}

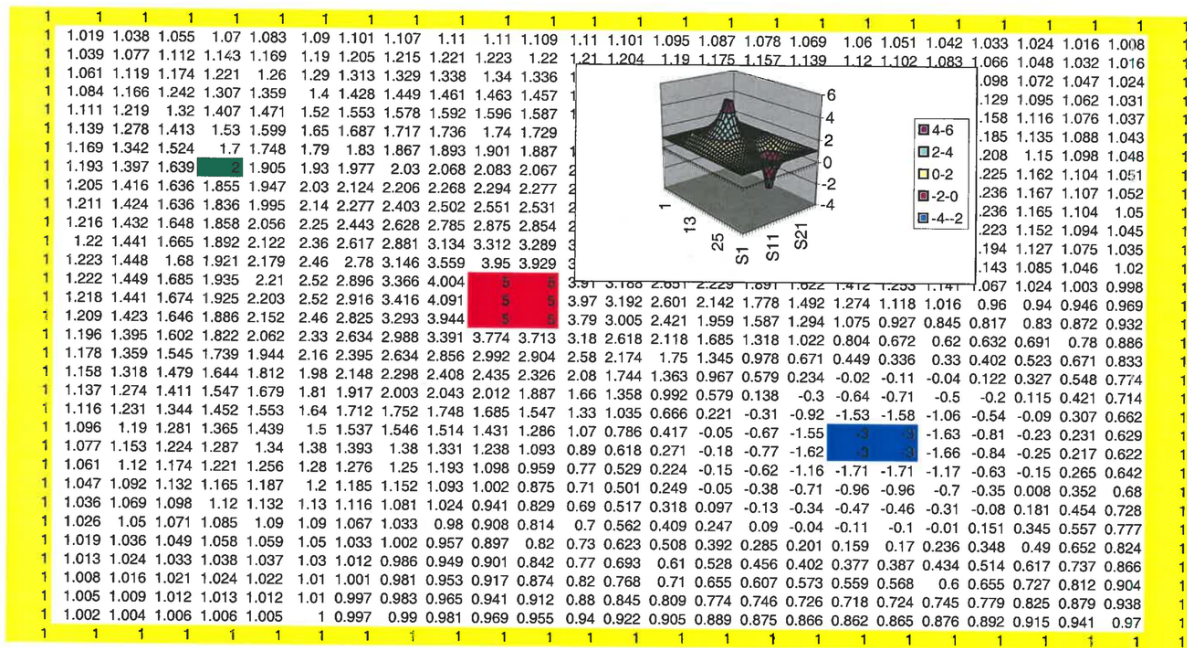
- e) Watch the numbers "relax" to the correct potential!

- f) Plot the results and see if it makes sense.

{Go to the "Insert" menu, click on "Chart", and select "Surface", then click "Next". You may have to select the area of the spreadsheet you wish to plot before doing this. If the chart looks good, just click "Finish". You can change the view of the surface by clicking on the chart with the right mouse button and then selecting "3-D View"} }



An example of the resulting spreadsheet of relaxed potential values, with the surface chart inset, is shown below.



g) Calculate the electric field by taking minus the gradient of the potential using finite differences:

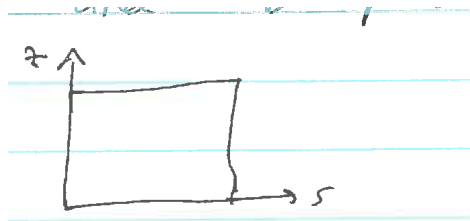
$$E_x = -\frac{\partial V}{\partial x} \approx -\frac{V(x + \Delta x) - V(x)}{\Delta x} \quad \text{etc.}$$

{Go to sheet2 and enter at cell C3: =Sheet1!D3-Sheet1!C3} to get E_x

{Go to sheet3 and enter at cell C3: =Sheet1!C4-Sheet1!C3} to get E_y

{Go to sheet4 and enter at cell C3: =SQRT((Sheet2!C3)^2+(Sheet3!C3)^2)} to get $|\vec{E}|$

h) Three-dimensional problems with cylindrical symmetry can also be solved by treating only the s-z plane and using the Laplace's equations in cylindrical coordinates.

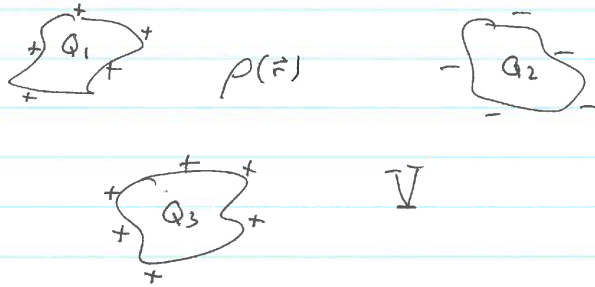


Many electrostatic problems come in this form:

- Specify the charge on a series of conductors
- Specify $\rho(\vec{r})$ in the region in between the conductors

The potential is not known on these conductors, so Uniqueness Theorem #1 may not apply:

So Uniqueness Theorem #1 may not apply:



Now solve for V : $\nabla^2 V = -\rho/\epsilon_0$ with these charged conductors. Is there a unique solution for $\vec{E}$?
Yes.

Uniqueness Theorem #2

In a volume $\mathcal{V}$ containing conductors and a specified charge density ρ , the electric field is uniquely determined if the total charge on each conductor is given.

The volume $\mathcal{V}$ can be bounded by another conductor, or be unbounded.