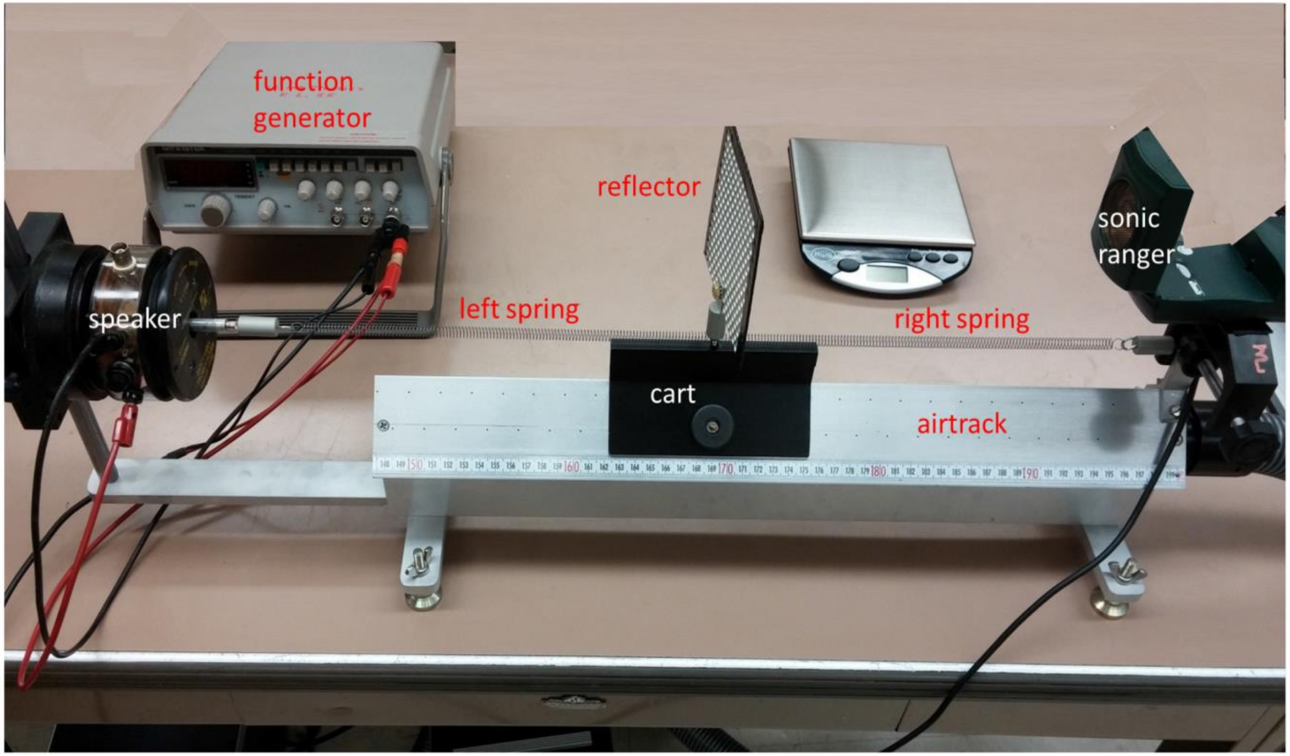
PHYS 275 – Experiment 10
Forced Harmonic Motion

Experiment Summary

- Today we will study a forced harmonic oscillator
  - Review harmonic motion, and what happens under a driving force



Today's Goal

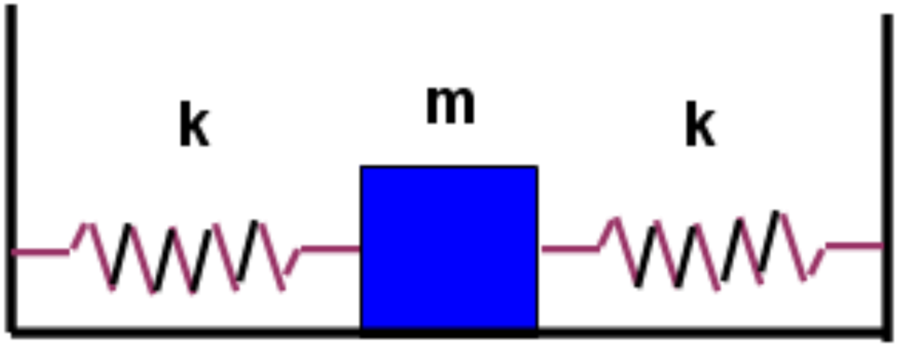
- Learn how a driven mass-spring system works
- Characterize a resonant behavior
  - What is a resonance? What is the quality factor of an oscillator?
- Estimate experimental parameters
  - How can I estimate the quality factor? How can I estimate the damping time?

- Today we will use an excel template
  - In the class document folder, it is the file that starts with "Copy…"; open the file, fill the colored cells, then save a copy, which will appear in the "My Documents" folder

Review of Harmonic Motion

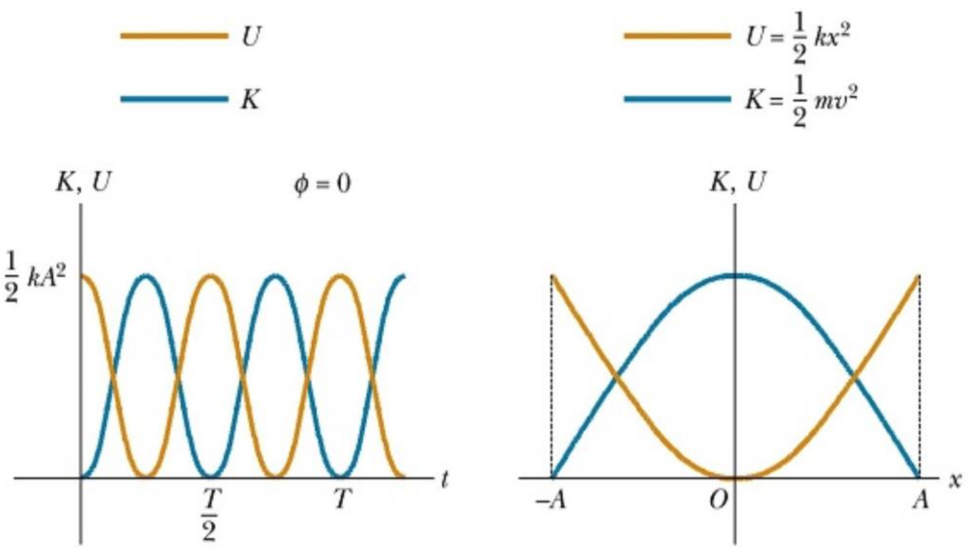
- Hooke's Law: $F = -kx$
  - Ideal spring is massless, and linear: force F is proportional to the displacement x
    - Force measured in Newton; displacement in meters
- Harmonic motion: periodic motion in which the restoring force is directly proportional to the displacement, and acts in the opposite direction
  - E.g.: spring with constant k attached to mass m:
    - $F = m\frac{d^2x}{dt^2} = -kx \Rightarrow x(t) = A\cos(\omega_0 t + \varphi);\ \omega_0 = \sqrt{\frac{k}{m}}$

Our Experiment



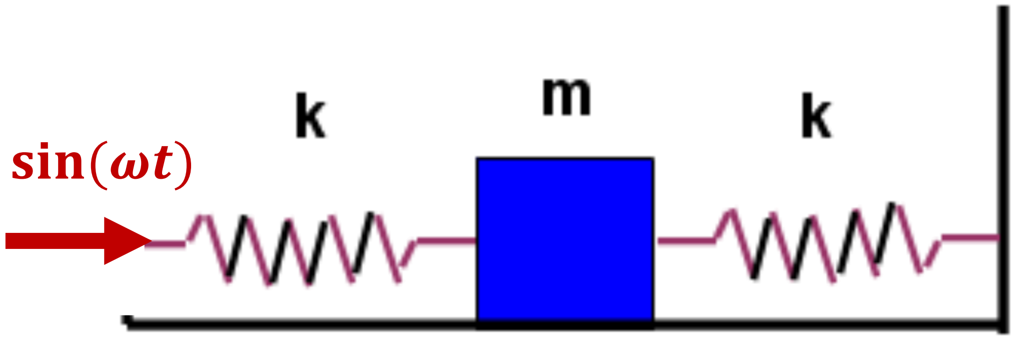
- $F = m\frac{d^2x}{dt^2} = -2kx$ (two springs) $\Rightarrow x(t) = A\cos(\omega_0 t - \varphi)$; $\omega_0 = \sqrt{\frac{2k}{m}}$
- How does the energy change?

The energy oscillates between kinetic (the mass moves) and potential (the springs are compressed/stretched)
Note that the total energy is fixed, if there is no damping



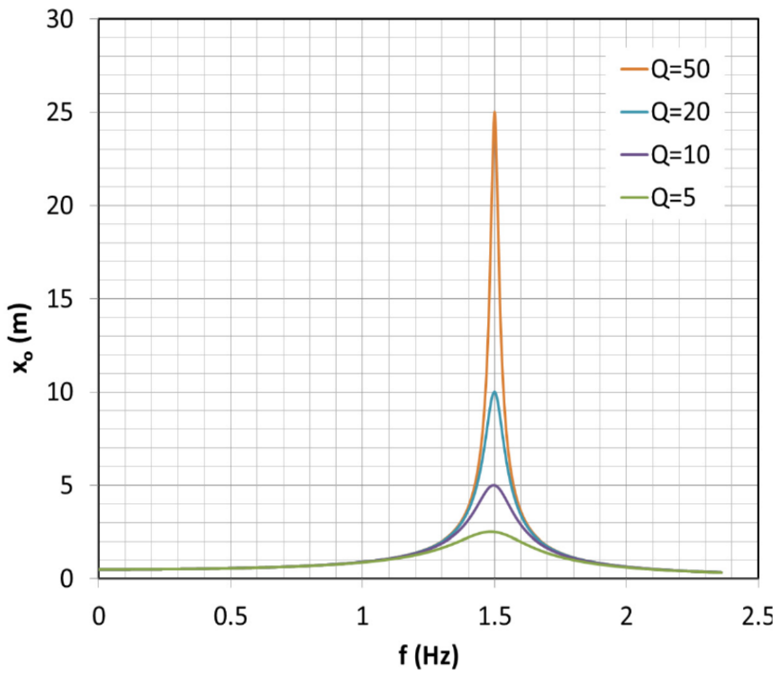
Adding a Driving Force

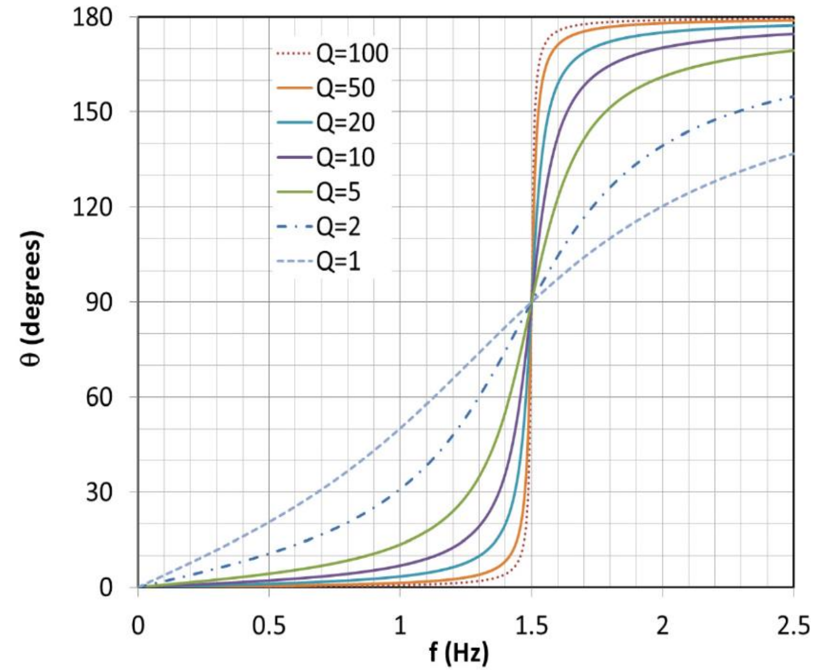
$F_{drive}(t) = kA_0 \sin(\omega t)$



- Now: $F = m\frac{d^2x}{dt^2} = -2kx + F_{drive}(t)$
  - Steady-state solution (after we let the oscillator settle a couple of minutes): $x(t) = x_0 \cos(\omega t - \theta)$
    - Note: frequency is not ω_0 anymore, it is the frequency of the driving force!
    - $x_0 = \frac{A_0/2}{\sqrt{\frac{\omega^2}{Q^2\omega_0^2} + \left(\frac{\omega^2}{\omega_0^2} - 1\right)^2}}$; $\theta = \tan^{-1}\left[\frac{1}{Q}\left(\frac{\omega\omega_0}{\omega_0^2 - \omega^2}\right)\right]$

Resonance

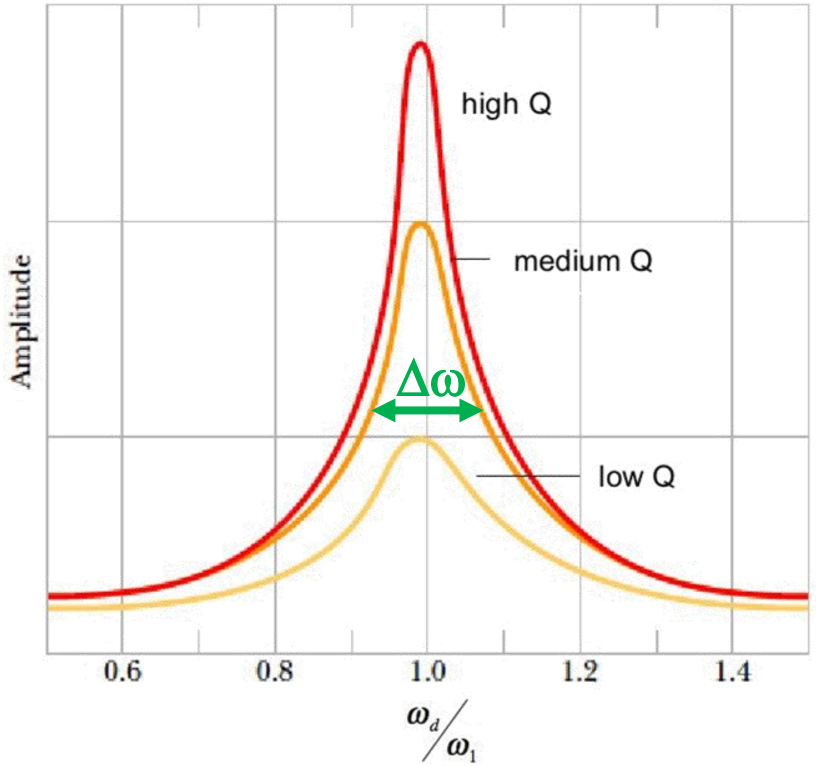




- x_0 is max at a special frequency: <u>resonance</u>
  - $\omega_{resonance} = \omega_0\sqrt{1 - \frac{1}{2Q^2}}$ … what is Q?
  - Note that also θ gets a special value: $\pi/2$

Quality Factor

- Q is a dimensionless parameter that characterizes the "quality" of the oscillator
  - It is defined as the ratio between the energy stored (kinetic + potential) and the energy lost per radian of oscillation
    - One full oscillation = 2π radian
  - It also characterizes the resonator's bandwidth relative to its center frequency
    - See plot: the higher Q, the narrower the resonant band
- The larger Q, the more slowly energy is lost, and the more slowly the oscillations die out



Suggestions and Notes

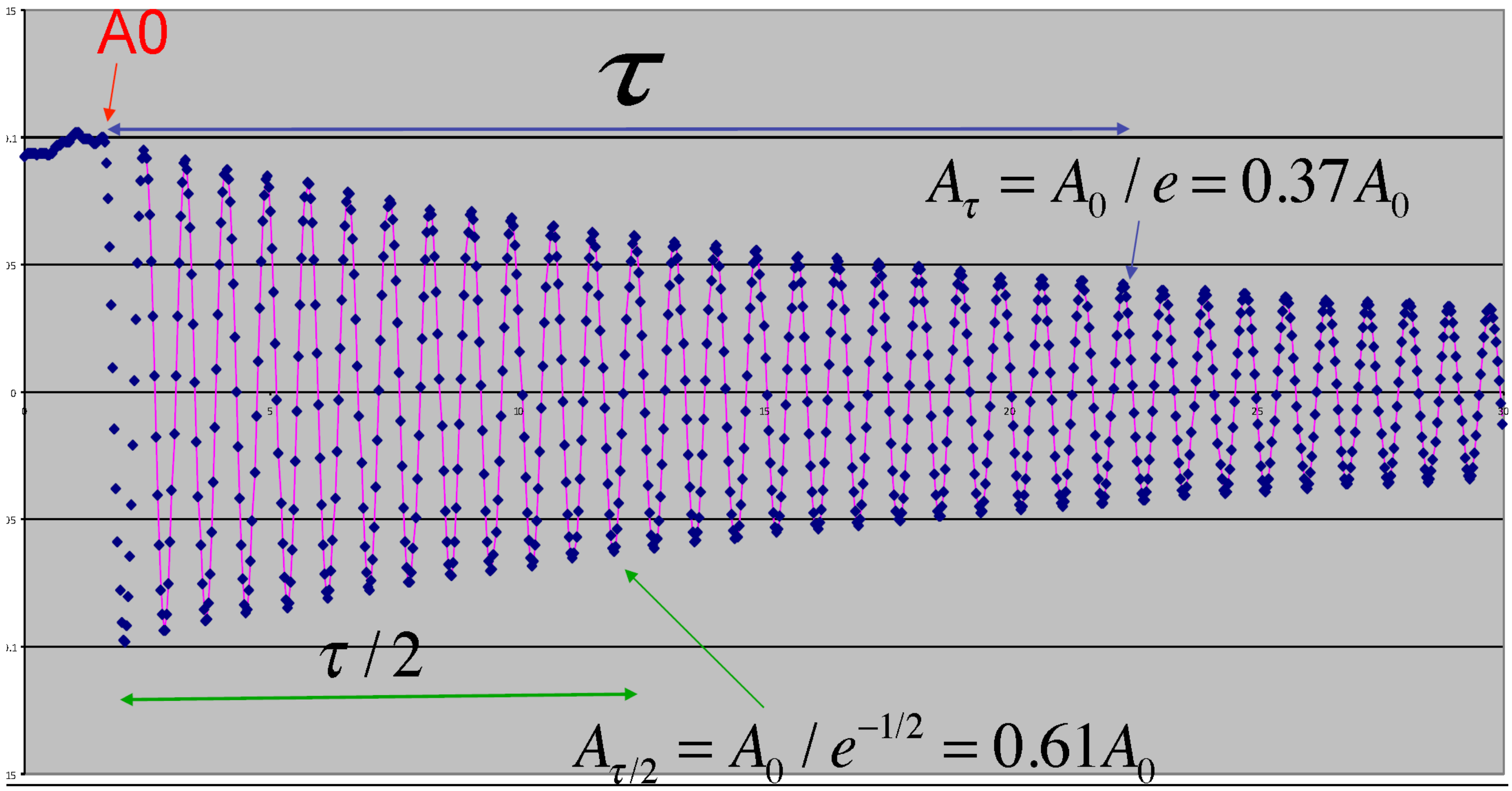
- You will be using an Excel template to perform all fits and data analysis
  - The file is in the LoggerPro Experiments folder, the name starts with "Copy…"
- The template is not very flexible
  - Make sure to follow to the letter the instructions in the manual! E.g., the SonicRanger sampling rate and sampling time
- Once you open the template, you need to fill in entirely before you will be able to save a copy (or leave Excel…)
  - Start by putting your name and section number; make up a name for your lab partner
  - There is one "Capture Image" function: DO NOT USE IT (not needed for our lab)
- Make sure your fits have no errors
  - If the fit macro fails, take the data again & re-do the fit until successful

More Suggestions

- How to optimally use the function generator
  - Make sure that the amplitude current is maximal
  - Adjust the frequency using both the coarse and fine knobs
    - Be patient: it takes a few seconds for the frequency to settle
- Check the speaker to verify that the membrane is oscillating
  - When the function generator is on…
- Even after you zero-ed the SonicRanger, there may still be a residual offset
  - Take some data (10 seconds?) with the air pump off and the cart in its rest position, take the average of your measurements and indicate it as your x_{offset} in the spreadsheet

Last Suggestion

- How can we estimate the oscillation frequency and the damping time?



Notes and Reminders

- Submit your Excel spreadsheet on ELMS and turn in your check sheet <u>before</u> leaving the lab
- Complete the final version of your report by 1pm next week
- Finish the homework set in Expert-TA by 2pm next week
- Turn off your equipment and clean up your bench area before leaving the classroom
- <u>Save your data on the local disk frequently!</u>