Homework #9 - Phys 273

1) String 1 and string 2, of mass densities (ρ1) and (ρ2) and tension (T), are connected together. Consider a wave traveling in string 1 towards the boundary with string 2. Find the ratio of the reflected amplitude to the incident amplitude, and the ratio of the transmitted amplitude to the incident amplitude, for the cases ρ2/ρ1 = 0, 0.25, 1, 4, and ∞.

2) Consider again two strings of mass density (ρ1) and (ρ2) and tension (T) connected together. Show that when a traveling wave strikes the boundary, the sum of the energies carried by the reflected wave and transmitted wave is equal to the energy carried by the incident wave.

3) Consider a single string of uniform mass density (ρ) and tension (T). In the center of the string, at x = 0, is a small lead ball of mass (M). The lead ball causes incident travelling waves to be partially reflected and partially transmitted. We would like to calculate the amplitudes and relative phases of the reflected and transmitted waves.

So consider an incident wave travelling in the positive (x) direction. It is described by

$$y_i(x,t) = A_1 e^{-i(kx-\omega t)}$$

There are also reflected and transmitted waves described by

$$y_r(x,t) = B_1 e^{-i(-kx-\omega t)}$$
$$y_t(x,t) = A_2 e^{-i(kx-\omega t)}$$

The boundary conditions are that

1) the incident and reflected waves must equal the transmitted wave at x = 0, and
2) the difference in the transverse force due to the waves on the left and the waves on the right must be equal to the mass of the lead ball times its acceleration. (This is Newton's 2nd Law for the lead ball.) The transverse force for a wave is given by

$$F_T = -T\frac{\partial y}{\partial x}.$$

a) Show that the first boundary condition implies that $A_1 + B_1 = A_2$.

b) Show that the second boundary condition implies that $A_1 - B_1 = \left(1 + \frac{iM\omega^2}{Tk}\right)A_2$.

c) Show that $\frac{A_2}{A_1} = \frac{1}{1+iq}$, where $q \equiv \frac{M\omega}{2Z}$, and Z is the impedance of the string.

d) Show that $\frac{B_1}{A_1} = \frac{-iq}{1+iq}$.

e) Suppose that $M = 0.10$ kg, $T = 100$ N, $\rho = 0.25$ kg/m, $A_1 = 0.05$ m, and $\omega = 200$ Hz. What is the amplitude of the transmitted wave, and what is its phase shift relative to the incident wave?

f) Repeat part (e) for the reflected wave.